THE DYNAMICS OF THE BRAZILIAN INCOME*

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Abstract: This paper aims to measure the degree of income mobility in Brazil in the 1987-2005 period. To achieve that, we consider the axiomatic mobility approach and the dynamic tool suggested by Aebi et al. (1999). The transition probability matrix calculations and the mobility index indicate that Brazil has low intragenerational income mobility, suggesting that Brazilian social structure is relatively rigid.
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1. INTRODUCTION

The high and persistent income inequality in Brazil has gained international notoriety. This is due to the fact that income concentration showed high and persistent levels between 1970 and 2000 after gathering strength in the 1960s. This places Brazil at the top of the world's income inequality ranking, giving the country a bad reputation with regard to earnings distribution.[1]

However, some recent changes have turned this trend around, characterizing an inflection point on the path of inequality measures.[2] In this regard, we have the direct and indirect effects of the Real Plan: a) inflation control and the resulting economic stability were key factors in the reduction of income concentration indices, since they created a favorable environment for the implementation of income transfer programs[3] and; b) impacts of trade liberalization and subsequent change in the structure of workforce qualification, with direct effects on earnings distribution.[4]

These characteristics have raised scientific and popular interest in earnings distribution in Brazil, calling for a specific study on this issue. Nevertheless, any strategy aimed at elucidating

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[1] See Neri (2006) and United Nations Development Program (2006).

[2] This change can be seen after 2001, when the indices dropped to the lowest levels ever reported since the mid-1970s. For detailed information, visit the website of the Brazilian Institute for Applied Economic Research: http://www.ipeadata.gov.br.

[3] See Barros et al. (2001) and Neri (2006).

[4] See Figueiredo et al. (2007).

earnings distribution should contemplate two elements: a) the static component, associated with the level of inequality, usually gauged by concentration indices and; b) the dynamic component, related to the notion of "income mobility."[5] The distinction between these two components lays the ground for empirical research. It is common knowledge that most studies seek to investigate income distribution by relying upon the definition of inequality, without showing concern for its counterpart. However, discussions about the origin of income mobility, as well as efforts put in to measure it, have abounded in the economic literature (see, for instance, Fields (2001)).

Mobility can be defined as the evolution of inequality over time since, in practice, individuals and/or families constantly change their economic positions. This movement may be associated with several factors: business cycles, changes in the level of education, promotions, migration, divorces, among others.

As previously pointed out, an increasing number of studies have dealt with income mobility. Roughly speaking, the literature can be categorized into three research groups: a) the first one, known as "axiomatic" approach, is concerned with the formulation of indices and with the description of their properties. In this context, we should cite the studies by Shorrocks (1978), Bartholomew (1982), Geweke et al. (1986) and Fields and Ok (1996); b) the second group seeks to associate the dynamics of income inequality with economic welfare. The studies by Atkinson (1981), Atkinson and Bourguignon (1982), Dordanoni (1992) and Gottschalk and Spolaore (2002) are important references on this issue and; c) the third group consists of empirical investigations, which include a large number of studies and diverse methodologies, but are restricted to a small number of countries.[6]

Note that empirical investigation deserves special attention. The collection of dynamic information requires that a sample of individuals is observed at different periods in time (or at least at two periods). In other words, it is necessary that the data panel identify each person (or family) in a given period. Such requirement, coupled to the lack of data panels with such characteristics, made this field of research become systematically neglected by the Brazilian empirical literature.

Fortunately, some statistical approaches propose solutions to this setback.[7] All that they need is percentage information about individuals in each income class at distinct periods. Most estimation methods produce a Markov transition matrix, which generates a mobility index in the spirit of Shorrocks (1978).

Based on these facts, one may infer that research targeted at investigating income distribution in Brazil should contemplate both dimensions of this phenomenon. In a recent study, Figueiredo and Ziegelmann (2006) partially fulfilled this requirement. In brief, the authors used static tools and detected a statiscally significant change in earnings distribution in Brazil, characterized by an increase in the number of individuals at the more central area of the distribution comparatively to individuals at the lower and upper tails. This movement was

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[5] Income mobility can be better understood by analogy with a hotel, as drawn by Joseph Schumpeter: suppose that a given hotel has more luxurious bedrooms on the upper floors and poor-quality bedrooms on lower floors. Therefore, the higher the floor, the better the quality of the bedrooms. Also suppose that on arriving at the hotel individuals choose to stay on lower floors and that with time they move onto the immediately upper floor. Thus, inequality would be associated with the quality of floors and with their distribution among guests. Mobility is concerned with the degree at which individuals change floors over time.

[6] More specifically to the U.S.A. and Germany. We recommend the studies by Gottschalk (1997), Trede (1998), Morillo (1999) and Aebi et al. (2001).

[7] Most studies include the calculation of relative entropy except for Lee et al. (1977). See Adelman et al. (1994), Golan et al. (1996) and Aebi et al. (1999).

compatible with a higher level of economic welfare. Nonetheless, despite the importance of these results, the study does not measure mobility.

In an attempt to fill this gap, the present paper aims to measure income mobility in Brazil between 1987 and 2005. To do so, we use the axiomatic approach to income mobility and construct a Markov transition matrix by utilizing the dynamic tools developed by Aebi et al. (1999). Thereafter, we calculate the mobility indices described in Prais (1955) and Shorrocks (1978).

The rest of the paper is structured as follows. Section 2 presents the assumptions related to Markov properties. Section 3 lays out the inference methods. In Section 4 the empirical results are shown, whereas Section 5 presents the final remarks.

2. INCOME DISTRIBUTION: A DYNAMIC ANALYSIS

The main objective of a study on economic mobility is to measure welfare distribution over time. In this regard, four methodological aspects should be taken into account. Firstly, the data on economic units should be identified and monitored over time. Secondly, one should be able to apply the analysis to a wide variety of economic units. Usually, individuals or families are sampled. Thirdly, several welfare dimensions can be investigated, but the income dimension is the one most commonly used. Finally, studies focus on the comparison of the initial year with the final year.

These characteristics favor the use of Markov transition matrices as a tool for measuring economic mobility. However, their use implies a fundamental hypothesis: the evolution of income distribution over time will be governed by a first-order Markov process. Thus, earnings distribution will follow a stochastic process represented by a transition matrix which, under certain circumstances, will converge to an equilibrium regardless of the initial distribution. In this section, we present the major assumptions related to this model. To do that, we depart from hypothetical matrix A. This matrix represents income transition probabilities between two time periods (**I** and **II**):

$$A = \begin{pmatrix} 0.64 & 0.29 & 0.04 & 0.03 & 0.00 \\ 0.14 & 0.56 & 0.26 & 0.03 & 0.01 \\ 0.02 & 0.22 & 0.54 & 0.21 & 0.01 \\ 0.01 & 0.04 & 0.27 & 0.54 & 0.14 \\ 0.00 & 0.01 & 0.05 & 0.27 & 0.67 \end{pmatrix}.$$

The transition matrix serves as a basis for Markov chain models. The elements of A represent the probability of individuals belong to class i in year **I** and migrate to class j in year **II**, i.e., transition probability p_{ij}. Therefore, by looking at the first row of the matrix, we can say that the individual who was in the first income *quintile* at year **I** has the following transition probabilities towards year J: 0.64 of staying at the same level; 0.29 of moving into the second class; 0.04 of moving up to the third class; 0.03 of ending up in the fourth class and; zero probability of reaching the top of the income distribution. A similar analysis can be carried out for the remaining rows of the matrix.

After establishing the framework for the Markov model, the following assumptions should be considered:

(S1) Population Homogeneity: the transition probability is the same for all individuals in the investigated income classes.

(S2) First-Order Markov Process: the current position of individuals at time m depends only on their immediately preceding past position at time $m-1$.

(S3) Time Homogeneity: the transition probabilities, p_{ij}, remain unchanged over time.

Therefore, the evolution of income can be described by $n(t_m) = n(t_{m-1})P$, where $n(t_m)$ represents the vector of marginal probabilities for each income class m periods after the beginning of the process. As previously mentioned, under these circumstances, the process will converge to a steady state, in such a way that the distribution of the equilibrium, $n*$, will not rely on the initial distribution $n(t_0)$.

The association between the Markov process and income distribution over time was developed by Champernowne (1953). From then on, this strategy has been widely referenced in the specialized literature.[8] Note that this approach is not the only alternative for investigating the dynamics of earnings distribution. Some non-Markov models can be found in the literature, such as that developed by Lydall (1974).[9]

In the following we describe the estimation method used to estimate transition matrices. This method and the difficulties surrounding its implementation will be dealt with in the subsequent section.

3. INFERENCE METHODS

The discussion in Section 2 exposes the basic theoretical features of Markov processes applied to the income evolution over time. It also provides the main argument in favor of using transition matrices as a tool for measuring economic mobility. Nevertheless, the latter topic deserves special attention, since the nature of the data does not always allow for the implementation of this strategy.

For instance, the analysis of the dynamics of Brazilian income stumbles upon some considerable setback: the Brazilian National Household Survey (PNAD), major source of data, does not provide information on each individual (or family) on a yearly basis. In other words, an individual in class i in the vector of the initial year (1987) would probably not belong to the sample in of the final year (2005). Even if it were in the final sample, we could not identify it. It is only possible to have percentage information on the number of observations within each income class in the several sampled years. This characteristic hinders the implementation of

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[8] With regard to personal income distribution, the following studies are of note: Shorrocks (1976), Gottschalk (1997) and Aebi et al. (2001). The study by Quah (1996) uses the Markov approach to investigate the process of income convergence across countries.

[9] In brief, the author suggests that the distinction between permanent and transitory income may invalidate some considerations about the model.

models based on conventional Markov transition matrices and it might have discouraged research into the dynamics of Brazilian income.

Fortunately, some alternative methods are available in the literature. The studies by Lee et al. (1977), Adelman et al. (1994) and Golan, Judge and Miller (1996) are relevant in this case. Recently, the tool proposed by Aebi et al. (1999) has been combined with the previous approaches, presenting at least one advantage: the ability to collect dynamic information from only two vectors over time. To do that, one should take for granted that the income transition probabilities between the two periods can be optimally estimated based on iterative criteria, so as to minimize the distance between the estimated and the "true" income transition process.

The optimization criterion is based on the calculation of relative entropy,[10] found at the fundamental hypothesis of statistical mechanics, as follows: the selected income transition process should represent the most likely alternative amongst all possible options.[11] The subsequent subsection will take a further look at the arguments presented herein and will give special attention to the construction of the Markov transition matrix. Subsection 3.2 will deal with the mobility indices described by Prais (1955) and Shorrocks (1978).

3.1. Income Dynamics Using Cross-Section Information

The aim of this subsection is to introduce the fitting method proposed by Aebi et al. (1999). Before doing that, the following initial assumptions are necessary: a) the incomes of N different individuals over time follow a sequence of discrete probability distributions $\{q_t\}$, with $t \in \{1, 2, ...\}$; b) the time evolution of this income distributions occurs through a Markov chain, with initial distribution q_0 and; c) each density q_t can be discretized into k partitions (income classes). Then, the sequence of k-vectors $\{(q_{1t}, q_{2t}, ..., q_{kt})'\}$ will have the following properties: $q_{it} \geq 0$ and $\sum_{i=1}^{k} q_{it} = 1$, with $t \in \{1, 2, ...\}$.

We assume that the classes income joint distribution between two periods t and s, s>t, can be represented by a two-dimensional function $F = (F_{ij})_{i,j=1,...,k}$. Here F_{ij} denotes the probability of an individual who belongs to class i at initial time (t) is in class j at final time (s).

In this context, F is a bivariate joint density of an unobserved stochastic process that represents the "history" of income distribution. That being said, we may assume that the income dynamics between two periods can be indirectly measured by the product between the probability transition matrix $P = (p_{it})_{i,j=1,...,k}$ and arbitrary initial distribution of individuals income class at time t, given by $J = (J_1, ..., J_k)'$. Thus, distribution F is defined as follows:

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[10] Usually, the entropy method is used when the data have some kind of limitation (incomplete observations, small sample size or misspecification of the data generating process). However, despite its importance, it has been underinvestigated in the econometric literature. Nevertheless, here we cite some examples: a) White (1982) develops a maximum likelihood estimator for the case of misspecification of the model; b) Kitamura and Stutzer (1997) propose a GMM-like estimator, but with relative robustness for small samples and; c) Golan et al. (1996) synthesize the use of entropies in several econometric fields (linear, nonlinear and dynamic models,).

[11] In this context, the measurement of the dynamics of income distribution will be equivalent to fitting cell probabilities for contingency tables, where only marginal distributions are observed. This physical mechanics problem has been widely investigated by statistical studies. For further details, see Aebi (1997).

$$F = diag(J)P\,, \qquad [3.1]$$

where operator $diag(\cdot)$ turns the $k \times 1$ vector into a $k \times k$ diagonal matrix. Usually, definition (3.1) is not compatible with distributions q_t and q_s, requiring an adjustment. Thus F-adjusted (F^{adj}) must satisfy the following ***initial and terminal restrictions***

$$q_t = F^{adj} i \text{ and } q_s = \left(F^{adj}\right)' i\,, \qquad [3.2]$$

where i represents a $k \times 1$ vector with all elements equal to one.

The fitting method consists in: a) computing the probabilities of observing each particular income transition process and; b) selecting the process whose probability of generating the particular observed configuration of classes distribution has the lowest speed of convergence to zero as the sample size increases. In other words, supposedly, there are infinite densities F, each of them associated with a probability of occurrence[12] and; an optimization criterion is used to select the "most probable" income transition. The probabilities are calculated using the maximum likelihood method. The selection of the most probable F should consider that the probability of observing a particular process converges to zero as the number of individuals tends to infinity $N \to \infty$. Thus, we have the large deviation principle, i.e., the selected F^{adj} should have the slowest convergence rate to zero in terms of probability, within the set of all two-dimensional distribution z.

Let us now have a look at the fitting method in more detail. As previously reported, the first step consists in determining the probability of observing a particular income transition configuration. Under the hypothesis that the incomes of N individuals are independent, this probability will be

$$\prod_{i,j=1}^{k} \left(J_i p_{ij}\right)^{\Gamma_{ij}}\,,$$

Where $\Gamma_{i,j}$ denotes how many persons starting in income class i in period t arrive in income class j in period s. We know that the income transition of N individuals between densities q_t and q_s can occur through several paths. These several possibilities are summarized by the following arrangement:

$$\binom{N}{\Gamma_{11}}\binom{N-\Gamma_{11}}{\Gamma_{21}}\binom{N-\Gamma_{11}-\Gamma_{21}}{\Gamma_{31}}\cdots\binom{N-\sum_{j=1}^{k}\Gamma_{1j}}{\Gamma_{21}}$$

[12] This assumption is confirmed by Csiszár (1975), who observed that the set of two-dimensional distribution that satisfy (3.2), dubbed z, contains infinite elements.

$$\binom{N-\sum_{i}^{k-1}\sum_{j=1}^{k-1}\Gamma_{ij}-\sum_{j=1}^{k-1}\Gamma_{kj}}{\Gamma_{kk}}=\frac{N!}{\prod_{ij=1}^{k}\Gamma_{ij}!}.$$

Thus, the probability of following a particular path Γ will be calculated using the following formula:

$$P_N(\Gamma\,|\,diag(J)P)=\frac{N!}{\prod_{ij=1}^{k}\Gamma_{ij}!}\prod_{i,j=1}^{k}\left(J_i p_{ij}\right)^{\Gamma_{ij}}=N!\frac{\prod_{i,j=1}^{k}\left(J_i p_{ij}\right)^{\Gamma_{ij}}}{\Gamma_{ij}!}. \qquad [3.3]$$

After calculating the probabilities, we must now select the income transition that is closest to the “true” process. To do that, we use a fundamental hypothesis of statistical mechanics:[13] the selected two-dimensional density will represent the “most probable” income transition process amongst all densities belonging to z . Considering this principle is equivalent to minimizing the convergence of (3.3) to zero. i.e., minimizing[14]

$$\lim_{N\to\infty}\frac{1}{N}\log P_N(\Gamma\,|\,diag(J)P)=-H(y\;|\,diag(J)P), \qquad [3.4]$$

where $y=(y_{i,j})$ denotes matrix $\Gamma/N=(\Gamma_{ij}/N)$. Function $H(y\;|\,diag(J)P)$ stands for the relative entropy for the two-dimensional distribution y with respect to $diag(J)P$, and is defined by

$$H(y\;|\,diag(J)P)=\sum_{i,j}^{k}y_{ij}\log\left(\frac{y_{i,j}}{JP}\right). \qquad [3.5]$$

Ellis (1986) demonstrates that $H(y\;|\,diag(J)P)$ is a non-negative and strictly convex function. Note that (3.5) has an infimum equal to zero if $y=diag(J)P$. Thus, relative entropy measures the distance between the estimated $diag(J)P$ and unobserved y processes. Therefore, the optimization process consists of the minimization of (3.5), being subject to continuity restrictions (3.2). The Lagrangian for this problem will be

$$L=\sum_{i,j}^{k}y_{ij}\log\left(\frac{y_{i,j}}{JP}\right)-\sum_{i,j}^{k}l_{i,t}\left(\sum_{i,j}^{k}y_{ij}-q_{i,t}\right)-\sum_{i,j}^{k}l_{i,s}\left(\sum_{i,j}^{k}y_{ij}-q_{i,s}\right). \qquad [3.6]$$

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[13] For further details, see Chapter 1 in Ellis (1986).
[14] For further details, see Chapter 1 in Golan et al. (1996).

In (3.6), $l_{i,t}$ and $l_{i,s}$ are 2k Lagrangian multipliers associated with restriction (3.2). According to Corollary 3.3 proposed by Csiszár (1975), the problem will have a solution if at least one of the income transition processes satisfies restriction (3.2). The strict convexity of the relative entropy warrants the existence of a unique solution.

The optimal solution is obtained from the differentiation of (3.6) in relation to $y_{i,j}$. By equaling the first-order condition to zero, we obtain:

$$F^{adj} = \Phi_t F \Phi_s , \qquad [3.7]$$

where $\Phi_t = diag(f_{1,t},...,f_{k,t})$ and $\Phi_s = diag(f_{1,s},...,f_{k,s})$ correspond to the exponentials of Lagrangian multipliers associated with the initial and terminal conditions. In quantum mechanics, these elements are known as Schrödinger multipliers.[15] Note that, if all multipliers are equal to one, there will be no fitting, indicating that F satisfies (3.2).

Schrödinger multipliers can be obtained from the differentiation of (3.6) in relation to $l_{i,t}$ resulting in the Schrödinger system:[16]

$$f_{it} J_i \sum_{j=1}^{k} p_{ij} f_{is} = q_{it}$$
$$\left(\sum_{i=1}^{k} f_{it} J_i p_{ij} \right) f_{is} = q_{js} ,$$

Yielding to

$$P^{adj} = \tilde{\Phi}_s^{-1} P \Phi_s ,$$

where $\tilde{\Phi}_s = diag(\tilde{f}_{1,s},...,\tilde{f}_{k,s}) = diag\left(\sum_{j=1}^{k} p_{1j} f_{js} ,..., \sum_{j=1}^{k} p_{kj} f_{js} \right)$, with $P = (p_{ij})$. Note that the fitting of matrix P will only depend on the multipliers related to the terminal condition. Expression (3.8) contains the dynamic information on income for the study period and its analysis is based on that of traditional Markov matrices.

3.2. Mobility Indices

According to Shorrocks (1978), the mobility index corresponds to a real function $M(\cdot)$, defined from the set of transition matrices $\mathcal{P}$. From now on some axioms are imposed.

(N) Normalization: $0 \leq M(P) \leq 1$, $\forall P \in \mathcal{P}$.

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[15] See Aebi and Nagasawa (1992) and Aebi (1996).

[16] This system is solved using an iterative computational criterion called Iterative Proportional Fitting Procedure (IPFP).

(M) Monotonicity: $P \succ P' \leftrightarrow M(P) > M(P')$.

(I) Immobility: $M(I) = 0$.

(PM) Perfect Mobility: $M(P) = 1$, if $P = ux'$, where $u = (1,...,1)'$ and $x'u = 1$.

The first axiom restricts the variation of the index to the interval [0; 1]. The second axiom associates the characteristics of the transition matrix? the mobility index. That is, if a matrix P has a larger mobility than a matrix P', it will be socially preferable ($\succ$) and its index will be necessarily higher. In other words, since the probability of movement across income classes is represented by the elements outside the main diagonal of the transition matrix, then if $p_{ij} \geq p'_{ij}$, $\forall i \neq j$ and $p_{ij} > p'_{ij}$ for any $i \neq j$, the mobility indices for the matrices will be: $M(P) > M(P')$.

The latter two axioms represent two extreme situations. In the first case, we have a static society represented by an identity matrix. So, there is no mobility across income classes. The opposite situation occurs in perfect mobility, represented by a matrix P, necessarily with identical rows.

Some indices are presented based on these axioms, with a special focus on the measure proposed by Prais (1955):

$$M_P = \frac{r - tr(P)}{r - 1},$$

where $tr(\cdot)$ represents the matrix trace and r stands for its rank.

However, Shorrocks (1978) raises the following question: how can we make comparisons between matrices with different periods? That is, in order for comparisons across mobility indices to be coherent, the index must be dissociated from the effect of time (T). Thus, it is possible to carry out the analysis without worrying about the size of the interval between the two time periods (Δ_t). To do that, the author introduces a new axiom:

(TI) Time Invariance: $M(P;T) = M(P^{\Delta_t}; \Delta_t T),\ \Delta_t > 0$.

The index does not depend on a particular observation over time, since it is compensated for by the size of the interval used for the construction of the transition matrix. Two indices are compatible with the new axiom:

$$M_D = 1 - \left|\det(P)\right|^{a/T},\ a > 0, \qquad [3.10]$$

where $\det(P)$ corresponds to the determinant of the transition matrix P. The second measure is represented by:

$$M_L = 1 - \left|q_2\right|, \qquad [3.11]$$

where q_2 is the second eigenvalue of matrix P.

Theorem 1 proposed by Geweke et al. (1986) warrants that indices (3.10) and (3.9) will be compatible with axiom structures **N**, **M**, **I**, **PM** and **TI**. To achieve that, the eigenvalues of P must all be real and non-negative.

Another important characteristic of the matrix can be measured by:

$$h = -\frac{\log 2}{\log |q_2|},$$

i.e., by the speed of convergence of the calculated matrix to the Markov chain with an equilibrium distribution. Alternatively, h can be interpreted as a half life of the transition process. Intuitively, a rigid structure (low mobility) is associated with a slow convergence process, but the opposite occurs in case of perfect mobility.

In summary, these indices allow measuring income mobility using transition matrices. Note that the alternatives shown herein are valid for discrete processes. Geweke et al. (1986) extend these results to continuous-time Markov processes. This alternative is not within the scope of this study, though.

4. RESULTS

4.1. Data and Implementation of the Optimization Process

This subsection aims to discuss the nature and manipulation of data and to describe the major strategies related to the optimization process implemented in the study. “Family income,”[17] based on the Brazilian National Household Survey (PNAD) conducted by the Brazilian Institute of Geography and Statistics (IBGE), was used as variable, using the month of September of the respective years as reference. The first step consisted of currency conversion and deflation.[18] To obtain that, we used the procedure suggested by Corseuil and Foguel (2002).

Two considerations are necessary: a) the concept of family income and; b) family size adjustment. Family income was regarded as the sum of all earnings received by the individuals living in the same household. After that, the sample was adjusted for family size. The adjustment was based on the following rule: $R_{adj} = R_d / n^e$, where R_{adj} is the adjusted income; R_d is the household income; n is the number of individuals in the household, and e is the elasticity of family size. Parameter e is related to the existence of economies of scale.[19] An intermediate value was considered for elasticity ($e = 0.5$), following the recommendation by Atkinson et al. (1995).[20] Only the positive incomes were included, and the outliers (adjusted incomes greater than 50,000 Reais) were left out.

The analysis of income transition is carried out using two time periods. In this study, they correspond to 1987 and 2005. The necessary information for the estimation is summarized in the

[17] Several studies use this variable as object of analysis, among them we have: Jenkins (1995), Burkhauser et al. (1999) and Aebi et al. (2001).

[18] All of the values are denominated in Reais as of January 2005.

[19] Consider two extreme cases: a) $e = 1$ there are no economies of scale and; b) $e = 0$ there are economies of scale, i.e., an infinite number of individuals can live equally well in a given household.

[20] Note that other values have been tested for e. However, no significant changes occurred in the results.

vectors of the percentage of individuals per income class. There, partitions represent income *deciles* ($k = 10$), in which 1987 stands for the initial year.

The estimation of transition process F requires *a priori* specifications for J and P. After that, the optimization process initiates, with the use of the Iterative Proportional Fitting Procedure (IPFP) (see, for instance, Deming and Stephan (1940)), producing matrices F^{adj} and P^{adj}.

Let us assume $J = q_{1987}$, i.e., an arbitrary distribution equal to the relative frequency of individuals per income class at the initial year. The construction of matrix P was based on the following assumption: an individual can only move into an immediately higher or lower class once a year.[21] For example, a person who belongs to the second decile in 1987 will only move to the first or third decile in 1988. Matrices with this property are known as 3-band.[22] Therefore, the initial specification for the two-dimensional density will be: $F_1 = diag(q_{1987})P^{18}_{3-band}$.

4.2. The Dynamics of Income Distribution in Brazil

Table 4.1 shows the percentage of individuals per income decile for years 1987 and 2005. First, we can see that the "transition" between the two periods was favorable to the intermediate income class (3 through 8). This movement was followed by an increase on the average income (around 2.10%) and by the reduction in income inequality (Gini's coefficient). Figueiredo and Ziegelmann (2006) use static tools and confirm the statistical significance of this change and its compatibility with a better level of economic welfare. However, despite the importance of these results, what can we assure about income dynamics in this period?

The starting point for answering this question is established in Table 4.2, which represents the Markov transition matrix for 18 years of mobility in Brazil.

Table 4.1: Percentage of Individuals per Income *Decil*e

Income Deciles	**Years**	
	1987	**2005**
[1]	10.00	5.75
[2]	10.00	7.91
[3]	10.00	10.48
[4]	10.00	13.39
[5]	10.00	12.14
[6]	10.00	11.82
[7]	10.00	10.55
[8]	10.00	10.05
[9]	10.00	8.91
[10]	10.00	9.01
Average income	840.09	857.67
Gini's coefficient	0.577	0.542

Source: Research data.

Observe that the individual who was in the first *decil*e in 1987 has the following transition probabilities: 0.280 of staying at the same level; 0.307 of migrating to the second *decile*; 0.210

[21] An alternative can be found in Tauchen (1986).

[22] A matrix will be $(2y+1)$ **-band** if its elements $a_{ij} = 0$, when $|i - j| > y$.

of moving to the third decile; 0.121 of ending up in the fourth decile and; decreasing probabilities all lower than 0.05 after the fifth decile. Therefore, belonging to the poorest 10% at the initial year is a determining factor for not reaching the top of the distribution at the final year.

Table 4.2: Markov Transition Matrix – 1987-2005.

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
[1]	0.280	0.307	0.210	0.121	0.048	0.021	0.009	0.003	0.001	0.000
[2]	0.183	0.228	0.216	0.172	0.093	0.056	0.031	0.016	0.005	0.000
[3]	0.068	0.118	**0.190**	**0.214**	**0.151**	**0.113**	0.076	0.048	0.019	0.003
[4]	0.026	0.062	**0.141**	**0.201**	**0.171**	**0.149**	**0.114**	0.084	0.042	0.010
[5]	0.010	0.034	**0.102**	**0.175**	**0.173**	**0.165**	**0.140**	**0.114**	0.067	0.020
[6]	0.004	0.020	0.075	**0.152**	**0.165**	**0.173**	**0.152**	**0.135**	0.089	0.035
[7]	0.002	0.013	0.056	**0.129**	**0.155**	**0.169**	**0.160**	**0.150**	0.111	0.055
[8]	0.001	0.007	0.038	**0.102**	**0.134**	**0.159**	**0.159**	**0.163**	0.140	0.097
[9]	0.000	0.002	0.018	0.060	0.093	**0.124**	**0.139**	**0.165**	0.187	0.212
[10]	0.000	0.000	0.003	0.015	0.030	0.052	0.076	0.125	0.230	0.469

Source: Research data.

The behavior of the tenth decile is similar to that of the first one, but in an opposite fashion, that is, those who belonged to this class in 1987 have a small probability of migrating to lower classes. Except for the poorest 20% and the richest 20% ((1-2) and (9-10)), transition probabilities are always higher than 0.10 at the "middle" of the distribution (figures in boldface), indicating a favorable movement to intermediate income classes.

Some information related to the transition matrix is provided in Table 4.3. The first piece of information, represented by the relative entropy value, refers to the distance between the estimated and "true" processes. The value of 0.137 suggests goodness-of-fit, given that the infimum for this measure is zero (see formula (3.5)). The speed of convergence to the Markov chain with equilibrium distribution is relatively high. This is perceived by the observation of the "half life" value for the process ($h = 1.495$). According to Shorrocks (1978), a structure with perfect mobility has full convergence in only one period ($h \to 0$). Slower speeds of convergence are associated with large "half life" values ($h \to \infty$). Another important characteristic can be captured from the square of the second eigenvalue of matrix ($q_2^2 = 0.396$). For Theil (1972), this index represents "mobility imperfection".

Table 4.3: Information Related to the Transition Matrix

Information	Values
Relative Entropy	0.137
Half Life (h)	1.495
Index M_P	0.396
q_2^2	0.864
Index M_D	0.933*
Index M_L	0.371

Source: Research data. *$a = 1$.

Finally, we have the values for the mobility indices. We calculate measures (3.9), (3.10) and (3.11). The magnitude of these results is evident compared to international values. Table 4.4 shows some indices for industrialized and developing countries. Note that Brazil has one of the lowest mobility indices, being only superior to the Colombian mobility index.

Table 4.4: International Income Mobility

Countries	**Index** M_L
Chile	0.655
China	0.652
Peru	0.539
USA	0.478
Germany	0.473
Malaysia	0.373
Colombia	0.229

Source: Gottschalk (1997), Birchenall (2001) and Fields (2001).

This result indicates that Brazilian social structure still presents relative rigidity. In other words, the income class in which an individual is inserted will determine his/her future social position. Or equivalently, there is a large intragenerational dependence that shows how strongly the income of an individual at time t can influence his/her income at $t + 1$. For example, an economic agent belonging to the poorest 10% has a very low probability of moving up socially and reaching the upper income class.

Such behavior is coherent with the results related to intergenerational dependence, i.e., the role of parent's income in the determination of their child's income. This finding is corroborated by Ferreira and Veloso (2006), who use the PNAD data for 1996 and found low intergenerational mobility in Brazil. That is, parent's income tends to be transferred to their descendents in a greater magnitude than it is observed in industrialized countries.

However, the study by Figueiredo et al. (2007) demonstrates that, even at lower levels than those of industrialized countries, the increase in the Brazilian intergenerational mobility in the last few years is an undeniable fact. In brief, the authors measure this mobility based on the effect of parent's educational level on their child's educational level. Their results show a remarkable reduction in this influence between 1987 and 2003. In summary, educational mobility rose from 0.493 in 1987 to 0.550 in 2003, indicating that parent's level of education has an increasingly lower influence on their child's educational status.

Nevertheless, before stating a final judgement, we should highlight the following: the period selected for the construction of the transition matrix (1987 to 2005) is characterized by intense changes in domestic and external relations in Brazil. These changes can be summarized by inflation control and subsequent economic stability, favoring the implementation of income transfer programs, and by trade liberalization and the consequent change in workforce qualification and wages. The effects of these changes on static elements of income distribution have already been discussed by Neri (2006) and Figueiredo et al. (2007). It should be underscored that the use of such a heterogeneous period may bias the results for mobility.

In order to circumvent this problem, we estimate a transition matrix by considering only the period after the Real Plan (1995 to 2005). In this case, the two-dimensional density that triggers the optimization process will be: $F_2 = diag(q_{1995})P_{3-band}^{10}$. The results of this experiment are shown in Tables A.1 and A.2 in the Appendix. We find some changes in transition probabilities, a lower speed of convergence for the Markov chain with an equilibrium distribution and a greater mobility imperfection. Nevertheless, mobility indices, albeit lower than those shown in Table 4.3, did not change substantially, which indicates that the selected period has a negligible effect on the construction of the matrix.

Therefore, our conclusion is that Brazil has relatively rigid income mobility, both in the intergenerational and intragenerational spheres. Despite that, roughly speaking, the movement of economic agents occurs towards the intermediate income classes. This behavior is coherent with the static results described by Figueiredo and Ziegelmann (2006), Neri (2006) and Figueiredo et al. (2007). One of the key arguments of these studies is that this movement suggests some improvement in the distribution pattern and indicates that changes, although slow, are still underway, towards a higher level of social welfare. However, although mobility is part of this context, this conclusion cannot be carried over to dynamic results, given that the axiomatic approach used herein does not establish an explicit link with the theory of economic welfare.

5. FINAL REMARKS

The present study aims to measure income mobility in Brazil between 1987 and 2005. For achieving that, we use the axiomatic approach to mobility and estimate the Markov transition matrix and calculate the respective mobility indices. Due to database limitations, more specifically to the lack of information on each individual (or family) on a yearly basis, we choose to implement an inference method based on the calculation of relative entropy. The estimation process comprised two periods (1987-2005 and 1995-2005), as a way to filter out possible biases related to the changes observed in the first half of the 1990s (roughly speaking, the trade liberalization process and the implementation of the Real Plan).

Results suggest that Brazil has low intragenerational income mobility, indicating that its social framework is relatively rigid. In other words, the income class in which an individual is inserted will determine his/her future social position. This finding concerns both the estimation for the whole period (1987-2005) and the inference for the period after the Real Plan (1995-2005), indicating that the selected period has a negligible effect on the construction of the matrix.

As for the movement on income distribution, there is an increase at intermediate income classes in detriment of tail weights. This result is in line with static evidence, which shows this movement and also its influence on the rise of social welfare in recent times. However, even though income mobility is part of this phenomenon, the evidence found in this study is not enough to provide a formal link between income dynamics and the theory of welfare. In this regard, notwithstanding the importance of measuring mobility in Brazil, a question is left unanswered: is the mobility index measured by an axiomatic approach consistent with a higher level of economic welfare?

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APPENDIX

Table A.1: Markov Transition Matrix – 1995-2005.

	[1]	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**	**[7]**	**[8]**	**[9]**	**[10]**
[1]	0.357	0.293	0.195	0.088	0.038	0.018	0.008	0.002	0.001	0.000
[2]	0.247	0.232	0.214	0.133	0.078	0.049	0.029	0.014	0.004	0.000
[3]	0.100	0.129	0.203	0.180	0.138	0.108	0.076	0.045	0.019	0.002
[4]	0.039	0.070	0.155	0.174	0.162	0.147	0.118	0.082	0.045	0.008
[5]	0.016	0.039	0.113	0.153	0.166	0.165	0.147	0.112	0.072	0.017
[6]	0.007	0.024	0.084	0.134	0.159	0.173	0.160	0.133	0.096	0.030
[7]	0.003	0.014	0.063	0.113	0.150	0.170	0.170	0.148	0.120	0.049
[8]	0.001	0.008	0.043	0.090	0.130	0.160	0.169	0.161	0.152	0.086
[9]	0.000	0.003	0.020	0.053	0.091	0.126	0.149	0.165	0.205	0.188
[10]	0.000	0.000	0.003	0.014	0.031	0.055	0.083	0.127	0.257	0.430

Source: Research data.

Table A.2: Information Related to the Transition Matrix

Information	**Values**
Relative Entropy	0.117
Half Life (h)	1.685
Index M_P	0.860
q_2^2	0.439
Index M_D	0.930*
Index M_L	0.337

Source: Research data. $*a = 1$.