ROBUST MONETARY POLICY, STRUCTURAL BREAKS, AND NONLINEARITIES IN THE REACTION FUNCTION OF THE CENTRAL BANK OF BRAZIL

Gabriela Bezerra de Medeiros (UFPB)
Marcelo Savino Portugal (PPGE/UFRGS)
Edilean Kleber da Silva Bejarano Aragón (UFPB)

Resumo:
Neste trabalho, nós procuramos investigar a existência de não linearidades na função de reação do Banco Central do Brasil (BCB) decorrentes de incertezas desse policymaker acerca dos efeitos do hiato do produto sobre a inflação. Teoricamente, nós seguimos Tillmann (2011) para obter uma regra de política monetária ótima não linear que é robusta às incertezas acerca do trade-off produto-inflação na curva de Phillips. Além disso, nós realizamos testes de quebra estrutural para avaliar possíveis mudanças na condução da política monetária brasileira durante o regime de metas de inflação. Os resultados indicaram que: i) as incertezas acerca da inclinação na curva Phillips implicaram em não linearidades na função de reação do BCB; ii) não se pode rejeitar a hipótese de uma quebra estrutural nos parâmetros da regra monetária ocorrendo no terceiro trimestre de 2003; iii) houve um aumento na resposta da taxa Selic ao hiato do produto e uma redução da reação ao hiato da inflação corrente no regime Meirelles-Tombini; e iv) o BCB também tem reagido à taxa de câmbio durante o regime Meirelles-Tombini.

Palavras-Chave: Política monetária robusta · Regras de taxa de juros não lineares · Quebras estruturais · Endogeneidade · Brasil.

Abstract:
In this work, we seek to investigate the existence of nonlinearities in the reaction function of the Central Bank of Brazil arising from this policymaker’s uncertainties about the effects of the output gap on inflation. Theoretically, we follow Tillmann (2011) to obtain a nonlinear optimal monetary policy rule that is robust to uncertainty about the output-inflation trade-off of the Phillips Curve. In addition, we perform structural break tests to assess possible changes in the conduct of the Brazilian monetary policy during the inflation-targeting regime. The results indicate that: i) the uncertainties about the slope in the Phillips curve implied nonlinearities in the Central Bank of Brazil’s reaction function; ii) we cannot reject the hypothesis of a structural break in the monetary rule parameters occurring in the third quarter of 2003; iii) there was an increase in the response of the Selic rate to output gap and a weaker response to the current inflation gap in Meirelles Tombini’s administration; and iv) the Central Bank of Brazil has also reacted to the exchange rate in Meirelles-Tombini’s administration.

Keywords: Robust monetary policy · Nonlinear interest rate rules · Structural breaks · Endogeneity · Brazil

JEL Classification: E52 · E58
1 Introduction

In the 1990s, the inflation-targeting regime was adopted by several countries as an alternative for the conduct of monetary policy and for the maintenance of price stability. In Brazil, this regime was implemented by the Central Bank of Brazil (CBB) in July 1999. This happened six months after the exchange rate band system was replaced with a floating system. Given exchange rate overshooting and the rise in inflation and in inflation expectations, the Brazilian government intended to implement a policy regime that could maintain price stability and establish a new nominal anchor for inflation.

Numerous papers, seeking to assess the CBB’s monetary policy decisions during the inflation-targeting regime, have estimated the Taylor (1993) rule or the forward-looking reaction function introduced by Clarida et al. (2000). In line with Taylor’s (1993) monetary rule, the central bank adjusts the nominal interest rate to the deviations of current inflation from the inflation target and to the current output gap. On the other hand, Clarida et al.’s (2000) policy rule assumes the monetary authority adjusts the interest rate according to the inflation and output gap expectations for the future. Some authors like Minella et al. (2003) and Minella and Souza-Sobrinho (2013) estimated a forward-looking reaction function and concluded that the CBB had a strong reaction to inflation expectations. Sanches-Fung (2011) estimated reaction functions for the CBB in a data-rich environment. His evidence demonstrates that the CBB adjusted the Selic interest rate by following the Taylor principle, but that it did not respond systematically to exchange rate movements.

The papers referenced above take for granted that interest rate rules are linear functions of variables that indicate economic status. However, empirical evidence has pointed out important nonlinearities in the monetary policy rule. Nobay and Peel (2000), Schaling (2004) and Dolado et al. (2005) argue that a nonlinear optimal monetary rule takes shape whenever the central bank has a quadratic loss function and the Phillips curve is nonlinear. Bec et al. (2002), Nobay and Peel (2003), Dolado et al. (2004), Surico (2007), and Cukierman and Muscatelli (2008) mention that nonlinearities in the optimal monetary rule may be present should the monetary authority have asymmetric preferences for inflation and/or for the output gap. Kato and Nishiyama (2005) and Adam and Billi (2006) show that if the nominal interest rate has a lower bound equal to zero, the central bank may have a stronger reaction to a decrease in inflation so as to reduce the probability of deflation.

Brazilian studies on monetary policy rule nonlinearities look into specific characteristics of the CBB’s asymmetric reaction. For instance, Aragón and Portugal (2010), Sá and Portugal (2011), and Aragón and Medeiros (2013) describe an asymmetric preference of the Brazilian monetary authority for an above-target inflation during the inflation-targeting regime. Moura and Carvalho (2010) gather empirical evidence in favor of nonlinearities in the reaction function that is consistent with the CBB’s asymmetric preference for inflation. Lopes and Aragón (2014) evince that the nonlinearity in the interest rate rule results from time-varying asymmetric preferences, and not from possible nonlinearities in the Phillips curve. Schiffino et al. (2013) reveal the nonnegativity constraint on the Selic interest rate may hinder the calibration of the CBB’s preferences, being conducive to nonlinearities in the optimal monetary rule. Aragón and Medeiros (2014) estimate a reaction function whose parameters vary over time and conclude that the reaction of the Selic rate to inflation varies remarkably throughout the period, showing a downtrend during the inflation-targeting regime.

In contrast to the studies referenced above, this paper’s prime goal is to investigate nonlinearities in the CBB’s reaction function as a result of this policymaker’s concern about specification errors in the macroeconomic model. In particular, we follow Tillmann (2011) to obtain a nonlinear optimal monetary policy rule that is robust to uncertainties over the effects
of output gap on inflation. The estimation of this monetary rule allows checking for the presence of nonlinearities in the CBB’s monetary policy conduct produced by specification errors in the model. In addition, we seek to run structural break tests to assess possible changes in the CBB’s reaction function coefficients during the inflation-targeting regime. With regard to this latter goal, it should be borne in mind that, from 1999 to 2013, the Brazilian economy was assailed by several shocks (energy crisis in 2001, exchange rate crisis in 2002, recession in 2003, world economic crisis in 2008, etc.), and that the CBB was run by three different presidents (Arminio Fraga (1999-2002), Henrique Meirelles (2003-2010), and Alexandre Tombini (2011-). These facts may have altered the way the CBB reacts to macroeconomic variables, such as inflation and output.1

Because of the presence of endogenous regressors, the method used to test structural breaks in the CBB’s reaction function parameters will be the one developed by Perron and Yamamoto (2013). The procedures proposed by those authors are based on the estimation by ordinary least squares (OLS) and by instrumental variables (IV), which allows estimating structural break dates and running tests for checking whether these breaks are statistically significant.

This paper’s results can be summarized as follows. First, empirical evidence indicates the uncertainties about the slope in the Phillips curve implied nonlinearities in the CBB’s reaction function. More specifically, results suggest that when the output gap is positive, the response of the Selic rate to inflationary gap is increasing in relation to current inflation, and that it is stronger to positive inflationary gaps than to negative ones. Second, structural break test results reject the null of stability in the CBB’s reaction function parameters. In general, we noted the presence of a structural break in the third quarter of 2003. This indicates that the conduct of Brazilian monetary policy in Mr. Arminio Fraga’s administration was different from Henrique Meirelles’s and Alexandre Tombini’s. Third, there was an increase in the response of the Selic rate to output gap and a decrease in the reaction to current inflation gap in Meirelles-Tombini’s administration. Finally, empirical evidence suggests the CBB also reacted to the exchange rate in Meirelles-Tombini’s administration.

In addition to this introduction, this paper is organized into another four sections. Section 2 introduces the structural macroeconomic model and the monetary authority’s optimization problem, which serves as the theoretical basis for this study. Section 3 describes the empirical method used to estimate the CBB’s reduced-form reaction function and checks for the presence of structural breaks in the coefficients of this equation. Section 4 presents the results. Section 5 concludes.

2 Theoretical model

In this section, we introduce the structural macroeconomic model and the central bank’s optimization problem, and we also derive the optimal monetary rule that is robust to specification errors in the model.

2.1 The economic structure

The basic theoretical model upon which the present paper is framed is the standard new Keynesian model analyzed by Woodford (2003b), Galí (2008) and Tillmann (2011), among others. According to this model, the evolution of an economy is represented by the following two-equation system:

1 Evidence of the CBB’s unstable reaction function parameters has been provided by Lima et al. (2007) and by Aragón and Medeiros (2013, 2014).
where $y_t$ is the output gap (deviation of output from potential output), $\pi_t$ is the inflation rate, $E_t(y_{t+1})$ and $E_t\pi_{t+1}$ are the expected output gap values and inflation rate at $t+1$ conditional on the information available at $t$, $i_t$ is the nominal interest rate, $r^n_t$ and $u^n_t$ are, respectively, a natural interest rate shock and a cost shock. It is assumed that $r^n_t$ and $u^n_t$ are white noise processes with mean zero and variance equal to 1. Parameter $\beta \in (0,1)$ is the subjective discount factor and $\sigma^{-1} > 0$ is the intertemporal elasticity of substitution.

The IS curve, given by Eq. (1), is a log-linearized version of Euler equation for consumption derived from households’ optimal decision about consumption and saving, after imposition of the market clearing condition. The expected output gap value shows that, as households would rather smooth consumption over time, the expectation for a higher level of consumption leads to an increase in current consumption, thereby boosting the current demand for output.

The Phillips curve, given by Eq. (2), shows the behavior of overlapping nominal prices, through which firms have a constant probability of keeping output price fixed in any time period (Calvo, 1983). The discrete nature of price adjustment prompts each firm to mark up the price as future inflation expectation rises.

In line with Tillmann (2011), it is assumed that the monetary authority is uncertain over the coefficient that measures the slope in the Phillips curve, $\kappa_t$, given by:

\[
\kappa_t = \bar{\kappa} + z^\kappa_t
\]

Specifically, the central bank is cognizant of the reference value of this coefficient, $\bar{\kappa}$, but incognizant of the model’s distortion, $z^\kappa_t$. It is also assumed that the central bank is unable to construct a probability distribution for $z^\kappa_t$.

2 The monetary authority’s optimization problem

Let us suppose that, conditional on the information available at the beginning of the period, the monetary authority attempts to choose the current interest rate $i_t$ and a sequence of future interest rates so as to minimize:

\[
E_0 \sum_{t=0}^{\infty} \beta^t L_t
\]

subject to equations (1) and (2). The central bank’s loss function at $t$, $L_t$, is given by:

\[
L_t = \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + \lambda_y (y_t)^2 + \lambda_i (i_t - i^*)^2 + \lambda_{di} (i_t - i_{t-1})^2 \right]
\]

where $\pi^*$ is the inflation target, $\lambda_y$ is the relative weight on output gap, and $\lambda_i$ and $\lambda_{di}$ are the relative weights attached to interest rate stabilization around an implicit target, $i^*$, and around the interest rate at $t-1$, $i_{t-1}$. The monetary authority is assumed to stabilize inflation around the...
inflation target, to keep the output gap closed at zero, and to stabilize the nominal interest rate around target $i^*$ and around the nominal interest rate at $t-1$.

Even though the central bank views models (1) and (2) with $z_t^κ = 0$ (or $κ_t = \bar{κ}$) as most likely, it worries about the specification errors of this benchmark model. As the monetary authority is unable to build a probability distribution for $z_t^κ$, it will try to follow a policy that is optimal to the worst possible result in the neighborhood of the benchmark model. In consonance with Hansen and Sargent (2008), the specification error $z_t^κ$ of the worst case is assumed to be chosen by a malicious agent who seeks to maximize the central bank’s loss function. The model in which the monetary authority seeks to choose $i_t$ so as to minimize loss function (5) and the malicious agent seeks to choose $z_t^κ$ which maximizes this loss, was designated by Hansen and Sargent (2008) as the worst-case model.

By minimizing the loss function in the worst possible model within a given set of models, the central bank determines its optimal policy by taking into account some specification error in the model. This is done explicitly by supposing that, depending on the degree of robustness, the central bank assigns a budget $ω$ to the malicious agent, who uses these resources to produce specification error $z_t^κ$. Therefore, the malicious agent’s budgetary constraint is given by:

$$E_i \sum_{t=0}^{∞} \beta^t \frac{1}{2}(z_t^κ)^2 \leq ω \quad (6)$$

When $ω$ is zero, we have a non-robust control problem, i.e., the optimal policy is not robust to the model’s specification error. Hence, to analyze the effects of the specification error, it is necessary to let $ω$ be a positive constant.

As remarked by Hansen and Sargent (2008), a robust monetary policy is one that solves the following min-max problem:

$$\min_{[\hat{κ}_t]} \max_{[\hat{κ}^κ]} E_i \sum_{t=0}^{∞} \beta^t \left[ (\pi_t - \pi^*)^2 + λ_\pi(y_t)^2 + λ(i_t - i^*)^2 + λ_i(i_t - i^*)^2 - θ_κ(z_t^κ)^2 \right] \quad (7)$$

subject to equations (1), (2) and (6). Supposing the policymaker and the malicious agent are unable to commit themselves to an optimal plan, optimization problem (7) is solved under discretion. This implies that the central bank and the malicious agent take the expectations of the future variables as given and choose, respectively, the interest rate and the current specification error, reoptimizing them every period. In this case, the Lagrangian of the optimization problem can be written as:

$$\min_{[\hat{κ}_t]} \max_{[\hat{κ}^κ]} L = \frac{1}{2} \left[ (\pi_t - \pi^*)^2 + λ_\pi(y_t)^2 + λ_i(i_t - i^*)^2 + λ_i(i_t - i^*)^2 - θ_κ(z_t^κ)^2 \right]$$

$$- \mu^\pi_t \left[ (y_t) - E_i(y_{t+1}) + σ^{-1}(i_t - E_i(σ_{t+1}) - r^*) \right]$$

$$- \mu_i^\pi_t \left[ π_t - βE_i(σ_{t+1}) - (κ + z^κ_t)(y_t) - u_t \right] \quad (8)$$

where $\mu^\pi_t$ and $\mu_i^\pi_t$ are the Lagrange multipliers associated with the IS curve and with the Phillips curve. Parameter $θ_κ ∈ [0, +∞[$ is inversely related to $ω$, and indicates the set of models available for the malicious agent. If $θ_κ$ has a low value (or if $ω$ is high), the monetary authority designs a policy that is robust to a wide range of specification errors, $z_t^κ$. Thus, a smaller value of $θ_κ$ indicates the central bank’s deeper concern over the robustness of its policy.

2.3 The optimal monetary rule
The first-order conditions of min-max problem (8) concerning $y_t$, $\pi_t$, $i_t$ and $z_t^\kappa$ are, respectively:

$$\lambda_i (y_t) - \mu_i^\kappa + (\kappa + z_t^\kappa)\mu_i^\kappa = 0$$  \hfill (9)

$$\pi_t - \pi^\kappa = 0$$  \hfill (10)

$$\lambda_i (i_t - i_t^\kappa) + \lambda_{\omega_i} (i_t - i_{t-1}) - \mu_i^\kappa \sigma^{-1} = 0$$  \hfill (11)

$$-\theta_k z_t^\kappa + \mu_i^\kappa (y_t) = 0$$  \hfill (12)

Substituting (10) into (12), we obtain $z_t^\kappa = (\theta_k)^{-1}(\pi_t - \pi^\kappa)(y_t)$. So, we can use (10) and (11) to eliminate the Lagrange multiplier in (9) and obtain the following optimal interest rate rule:

$$i_t = (1-\rho_1)\left[c_0 + c_1(\pi_t - \pi^\kappa) + c_2 y_t + c_3(\pi_t - \pi^\kappa)^2 y_t\right] + \rho_{\kappa} \lambda_{t-1}$$  \hfill (13)

where

$$c_0 = i_t^\kappa; \quad c_1 = \frac{\kappa \sigma^{-1}}{\lambda_1}; \quad c_2 = \frac{\lambda_2 \sigma^{-1}}{\lambda_1}; \quad c_3 = \frac{\theta_k \sigma^{-1}}{\lambda_1}; \quad \rho_1 = \frac{\lambda_{\omega_i}}{\lambda_1 + \lambda_{\omega_i}}$$

Note that the difference between monetary rule (13) and the Taylor (1993) rule concerns the inclusion of variable $(\pi_t - \pi^\kappa)(y_t)$ and of the interest rate smoothing term, $\rho_1 i_{t-1}$. In addition, the smaller the value of $\theta_k$ (i.e., larger preference of the central bank for robustness), the stronger the central bank’s reaction to variable $(\pi_t - \pi^\kappa)(y_t)$.

In view of the nonlinear structure of Eq. (13), the long-term reactions of the monetary policy instrument to the deviation of current inflation from the inflation target and from the output gap are given by:

$$\frac{\partial i_t}{\partial (\pi - \pi^\kappa)} = c_1 + 2c_2 (\pi - \pi^\kappa)$$  \hfill (14)

$$\frac{\partial i_t}{\partial y} = c_2 + c_3 (\pi - \pi^\kappa)^2$$  \hfill (15)

Eq. (14) shows that, when the output gap is positive, the reaction of the interest rate to the inflationary gap (or deviation of inflation from the inflation target) has two major characteristics: i) it is increasing compared to the current inflation; and ii) it is larger for positive inflationary gaps than for negative ones with the same magnitude (in absolute terms). So, as highlighted by Tillmann (2011), the central bank’s uncertainty over the slope in the Phillips curve introduces nonlinearity and asymmetry into the optimal interest rate rule. With regard to the response to the output gap, Eq. (15) indicates that a larger inflationary gap produces a stronger reaction to output by the monetary policy.

3 Method

In this section, we derive the reduced form for the robust interest rate rule to be estimated, in order to check for nonlinearities in the CBB’s monetary policy conduct resulting from the specification errors mentioned in Section 2. Moreover, we present the estimation methods and the structural break tests used to verify the stability of coefficients in the monetary rule and the data to be used in the study.

3.1 Reduced form of the monetary policy rule
For the sake of estimation, three changes are made to monetary rule (13). First, a random shock to the interest rate, \( \nu_t \), is included in this equation. Second, we consider a variable inflation target (\( \pi_t^* \)). This change is necessary because, between 1999 and 2004, the inflation targets, set by the Brazilian National Monetary Council (CMN), changed on a yearly basis. Third, the nominal interest rate at \( t-2 \) is inserted into the policy rule in order to minimize possible serial autocorrelation problems.\(^3\) By making these changes, the specification of the policy rule to be estimated is given by:

\[
i_t = d_0 + d_1 (\pi_t - \pi_t^*) + d_2 y_t + d_3 (\pi_t - \pi_t^*)^2 y_t + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \nu_t \tag{16}
\]

where \( d_\tau = (1 - \rho_1 - \rho_2) c_\tau, \tau = 0, 1, 2, 3 \). Coefficients \( d_1, d_2 \) and \( d_3 \) measure the short-term (long-term) reaction of the interest rate to the current inflation gap, to the output gap, and to the interaction of the squared inflationary gap with the output gap.

### 3.2 Structural break tests

Reaction function (16) assumes regression parameters are constant over time. However, after the inflation-targeting regime was adopted in 1999, the Brazilian economy has been hit by numerous shocks (energy crisis in 2001, exchange rate crisis in 2002, recession in 2003, world economic crisis in 2008, etc.), and the CBB was administered by three different presidents (Arminio Fraga, Henrique Meirelles, and Alexandre Tombini). These facts may have altered the conduct of the Brazilian monetary policy, i.e., they may have changed the CBB’s reaction to inflation rate and output gap movements. To verify that, this paper uses structural break tests to test the hypothesis of stability of interest rate rule parameters.

The econometrics literature on tests for structural break in parameters of a regression is comprehensive.\(^4\) Recently, some studies have considered the problem with running structural break tests on equations with endogenous regressors (i.e., correlated with errors). Hall et al. (2012) show that the minimization of a two-stage least squares criterion yields consistent estimators for break fractions. Perron and Yamamoto (2014) present a simple proof of Hall et al.’s (2012) results. Additionally, they demonstrate that all assumptions made by Bai and Perron (1998, 2003a), obtained with original regressors contemporaneously uncorrelated with errors, are satisfied. Therefore, Bai and Perron’s (1998, 2003a) results hold for equations with endogenous regressors.

As monetary rule (16) contains potentially endogenous regressors (inflationary gap and output gap), this paper follows in the steps of Perron and Yamamoto (2013) and utilizes an alternative procedure to test structural breaks in linear models with endogenous regressors. This procedure consists in ignoring the endogeneity of regressors and running the structural break tests based on the estimation of the structural equation by ordinary least squares (OLS). This method is justifiable for four reasons, namely: i) changes in the real parameters of the model imply changes in the probability limits of the OLS estimator; ii) the model can be reformulated in order for regressors and errors to be uncorrelated, thereby allowing the use of the empirical procedure and distribution limits of the structural break tests introduced by Bai and Perron (1998, 2003a); iii) as the regressors obtained by instrumental variables (IV) estimation have less quadratic variation than original regressors, a change in the real parameters leads to a deeper change in the conditional mean of the dependent variable in an OLS framework than in an IV framework; iv) application of the OLS approach leads to consistent estimates of break fractions and ameliorates the efficiency of estimates and test

---

\(^3\) This procedure was also adopted by Aragón and Portugal (2010) and Minella and Souza-Sobrinho (2013).

\(^4\) There is a large number of statistical and econometric studies on these structural break tests. For an excellent review of these studies, see Perron (2006).
power in several situations. In order to illustrate the estimation of structural breaks, let a multiple linear regression model with \( m \) breaks at \( \{ T_1, \ldots, T_m \} \) be expressed by:

\[
i = \bar{X}d + \nu
\]  

(17)

where \( i = (i_1, \ldots, i_p) \) is the dependent variable and \( \bar{X} = diag(X_1, \ldots, X_{m+1}) \) is a \( Tx(m+1)p \) matrix with \( X_i = (x_{T_{i+1}}, \ldots, x_{T_i})' \) for \( i = 1, \ldots, m+1 \), \( T_0 = 0 \) and \( T_{m+1} = T \). Note that each matrix \( X_i \) is the subset of the matrix of regressors corresponding to regime \( i \). Matrix \( \bar{X} \) is a diagonal block of the \( Txp \) matrix of regressors, \( X \), with the block being based on the set \( \{ T_1, \ldots, T_m \} \). Some or all regressors in \( X \) may be correlated with the errors. The vector \( d = (d_1', \ldots, d_m')' \) is an \( (m+1)p \) vector of coefficients and \( \nu = (\nu_1, \ldots, \nu_p)' \) is the vector of disturbances.

Let the real break dates be denoted by a superscript 0, i.e., \( \{ T^0_1, \ldots, T^0_m \} \) and \( \bar{X}_0 \) be a diagonal block of \( X \) according to \( \{ T^0_1, \ldots, T^0_m \} \) and \( d^0 \) be the vector of the actual parameter values. In addition, the actual break fractions are denoted by \( (\lambda^0_1, \ldots, \lambda^0_m) = (T^0_1/T, \ldots, T^0_m/T) \).

Therefore, note that the data-generating process (DGP) for (17) can be written as:

\[
i = \bar{X}_0 d^0 + P_{X_0} \nu + (I - P_{X_0}) \nu
\]

\[
= \bar{X}_0 [d^0 + (\bar{X}_0' \bar{X}_0)^{-1} \bar{X}_0' \nu] + (I - P_{X_0}) \nu = \bar{X}_0 d^* + \nu^*
\]

(18)

where \( \nu^* = (I - P_{X_0}) \nu \) and \( d^* = d^0 + (\bar{X}_0' \bar{X}_0)^{-1} \bar{X}_0' \nu \). Note that \( d^* \to d^0 \) and \( \bar{X}_0 \) is uncorrelated with \( \nu^* \). Thus, the OLS estimator, \( \hat{d}^* \), will be consistent for \( d^* \). So, the break dates can be estimated by minimization of the sum of squared residuals of the regression:

\[
i = \bar{X}d^* + \nu^*
\]

(19)

The estimated break dates are given by:

\[
(\hat{T}^*_1, \ldots, \hat{T}^*_m) = \arg \min_{\pi \in \Pi} SSR^*_T(T_1, \ldots, T_m)
\]

(20)

where \( SSR^*_T(T_1, \ldots, T_m) = (i - \bar{X}d^*)'(i - \bar{X}d^*) \) is the sum of squared residuals for block \( (T_1, \ldots, T_m) \), such that \( T_i - T_{i-1} \geq q \) where \( q \geq 0 \) is the minimum amount of observations a regime \( i \) should have. Perron and Yamamoto (2013) show that the estimates of break fractions \( (\hat{\lambda}^*_1, \ldots, \hat{\lambda}^*_m) = (\hat{T}^*_1/T, \ldots, \hat{T}^*_m/T) \) are consistent and have the same convergence rate as those obtained from the usual OLS approach with regressors that are uncorrelated with errors.

To check for the presence of structural breaks in the CBB’s reaction function, we follow Bai and Perron (2003a) and use two tests. The first one is the sup\( F_T \) test, whose objective is to test the null hypothesis of no structural break against the alternative hypothesis of \( m = k \) breaks. To calculate the test statistic, we are going to denote \( (T_1, \ldots, T_k) \) as the block such that \( T_i = [T_{\lambda_i}] \) \( (i = 1, \ldots, k) \), and \( R \) as a matrix such that \( (Rd^*)' = (d^*_1 - d^*_2, \ldots, d^*_k - d^*_{k+1}) \).

Define

\[
F_T(\lambda_1, \ldots, \lambda_k; p) = \frac{1}{T} \left( \frac{T - (k + 1)}{kp} \right) d^* R'(R\hat{V}(d^*)R')^{-1} R\hat{d}^*
\]

(21)

where \( \hat{V}(d^*) \) is an estimate of the covariance matrix \( \hat{d}^* \). The sup\( F_T \) statistic is given by:
\[ supF_T (k; p) = F_T (\hat{\lambda}_1, ..., \hat{\lambda}_k; p) \] (22)

where \((\hat{\lambda}_1, ..., \hat{\lambda}_k)\) minimizes the total sum of squared residuals. The asymptotic distribution of the sup\(F_T\) statistic relies on a trimming parameter, \(\varepsilon = q/T\).

The second test, called \(F_T(l+1|l)\), is aimed at testing the null hypothesis of \(l\) breaks against the alternative hypothesis of \(l+1\) breaks. For the model with \(l\) breaks, the break date estimates are obtained through a sequential procedure (Bai, 1997; Bai and Perron, 1998). Bai and Perron’s (1998, 2003a) strategy consists in testing for the presence of an additional structural break in each of the \(l+1\) segments. The test is applied to each segment containing observations \(\hat{T}_{i-1}\) to \(\hat{T}_i\) \((i = 1, ..., l+1)\). We reject the null of \(l\) breaks in favor of a model with \(l + 1\) if the global minimum of the sum of squared residuals (in all segments in which an additional break is included) is sufficiently smaller than the sum of the squared residuals of the model with \(l\) breaks.

The critical values for the sup\(F_T\) and \(F_T(l+1|l)\) tests are shown in Bai and Perron (2003b). However, when the correlation between regressors and errors changes between segments or the marginal distribution of regressors contains modifications (due to a change in the mean and/or variance of regressors), the limit distributions of those statistics differ from the ones described by Bai and Perron (2003b).\(^5\) In this case, Perron and Yamamoto (2013) demonstrate that the sup\(F_T\) and \(F_T(l+1|l)\) tests can exhibit small size distortions. Following their suggestion, the results will take into account the standard critical values and those obtained from the fixed regressor bootstrapping method of Hansen (2000).

Although the OLS-based method is suitable for various situations, we also estimate the break dates and run the tests for structural breaks based on IV. To do that, we assume there is a set of \(q\) variables \(z_t\) that can be used as tools. Let \(Z = (z_1, ..., z_T)\) denote a \(T \times q\) matrix. We seek to estimate the unknown break dates using observed variables \((i, X, Z)\). In this case, the relevant IV regression is given by:

\[ i = X'd + \tilde{\nu} \]

where \(X' = \text{diag}(\hat{X}_1, ..., \hat{X}_{m+1})\), \(\hat{X}_j = (\hat{x}_{t_1}, ..., \hat{x}_{T_j})'\), \(e \hat{X} = (\hat{x}_1, ..., \hat{x}_T)' = P_zX\) where \(P_z = Z(ZZ')^{-1}Z'\). The error term is \(\tilde{\nu} = (\tilde{\nu}_1, ..., \tilde{\nu}_T)'\) with \(\tilde{\nu}_i = \nu_i + \eta_i\), \(\eta_i = (x_i' - \tilde{x}_i)\delta_j\) for \(T_{j-1} + 1 \leq t \leq T_j\). The break date estimates are given by:

\[ (\hat{T}_1, ..., \hat{T}_m) = \arg \min_{\hat{T}_1, ..., \hat{T}_m} SSR_T (T_1, ..., T_m) \]

(24)

where \(SSR_T\) is the sum of squared residuals of regression (23) estimated by OLS and assessed on \((T_1, ..., T_m)\).

In practice, we use the following procedure:

i) we estimate the reduced forms of \((\pi_t - \pi_t^*)\), \(y_t\) and \((\pi_t - \pi_t^*)^2 y_t\) by OLS, we find the break dates sequentially and use the sup\(F_T(l+1|l)\) tests to verify the statistical significance of these changes;

ii) if the reduced forms are unstable, we obtain the predicted values of \((\pi_t - \pi_t^*)\), \(y_t\) and \((\pi_t - \pi_t^*)^2 y_t\) from each subsample delimited by the break dates estimated in step (i). If the reduced form is stable, the regressor is obtained by considering the whole sample;

iii) we estimate reaction function (16) replacing endogenous regressors with those produced in step (ii) and we use the \(F_T(l+1|l)\) statistic to test for the presence of structural breaks in these equations.

\(^5\) For structural break tests with changes in the marginal distribution of regressors, see Hansen (2000).
Perron and Yamamoto (2013, 2014) highlight that the IV procedure described above is efficient when compared to that of Hall et al. (2012) because it uses all the information from the sample. However, if the reduced forms are unstable, the change in marginal distribution of regressors in the estimated structural equation precludes the use of the critical values demonstrated in Bai and Perron (2003b) for the supFT and FT(l+1|l) tests. In this case, we follow Perron and Yamamoto (2013) and use the fixed regressor bootstrapping method of Hansen (2000).

4 Results

4.1 Data and unit root test

The estimation of the CBB’s nonlinear reaction functions described in Section 3.1 is based on monthly data for January 2000 to December 2013. The series is obtained from the Brazilian Institute of Applied Economics Research (IPEA) and Central Bank of Brazil websites. The dependent variable, $i_t$, is the annualized Selic interest rate accumulated monthly. This variable has been employed as the main monetary policy instrument under the inflation-targeting regime.

The inflation rate, $\pi_t$, is the inflation accumulated over the past 12 months, measured by the broad consumer price index (IPCA). The monthly inflation target ($\pi_t^*$) is obtained from the linear interpolation of annual targets.

The output gap, $y_t$, is measured by the percentage difference between the seasonally adjusted industrial production index and potential output. Here, there is an important problem, as potential output is an unobserved variable and, for that reason, it must be estimated. Therefore, the proxy for potential output is obtained as follows: by the Hodrick-Prescott (HP) filter, by a linear trend (LT) model and by a quadratic trend (QT) model. The output gap series built upon the different potential output estimates are denoted as $y_{HP,t}$ (HP), $y_{LT,t}$ (LT) and $y_{TQ,t}$ (QT).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Exogenous regressors</th>
<th>ADF(k)</th>
<th>ERS</th>
<th>MZ$_a^{GLS}(k)$</th>
<th>MZ$_t^{GLS}(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_t$</td>
<td>c, t</td>
<td>-3.309 (4)</td>
<td>3.690 $^{**}$ (4)</td>
<td>-25.40 $^{**}$ (4)</td>
<td>-3.558 $^{**}$ (4)</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>c</td>
<td>-1.909 (13)</td>
<td>3.325 $^{**}$ (13)</td>
<td>-8.432 $^{**}$ (13)</td>
<td>-2.022 $^{**}$ (13)</td>
</tr>
<tr>
<td>$\pi_t^*$</td>
<td>c</td>
<td>-3.225 $^{**}$ (0)</td>
<td>5.271 (0)</td>
<td>-5.866 $^{**}$ (0)</td>
<td>-1.657 $^{**}$ (0)</td>
</tr>
<tr>
<td>$y_{HP,t}$</td>
<td>-</td>
<td>-3.521 $^{**}$ (0)</td>
<td>1.418 $^{**}$ (13)</td>
<td>-18.75 $^{**}$ (0)</td>
<td>-3.036 $^{**}$ (0)</td>
</tr>
<tr>
<td>$y_{LT,t}$</td>
<td>-</td>
<td>-2.497 $^{**}$ (0)</td>
<td>3.073 $^{**}$ (0)</td>
<td>-9.305 $^{**}$ (0)</td>
<td>-2.070 $^{**}$ (0)</td>
</tr>
<tr>
<td>$y_{TQ,t}$</td>
<td>-</td>
<td>-2.792 $^{**}$ (0)</td>
<td>2.613 $^{**}$ (0)</td>
<td>-10.59 $^{**}$ (0)</td>
<td>-2.254 $^{**}$ (0)</td>
</tr>
</tbody>
</table>

Note: * denotes significance at 1%, ** at 5%, *** at 10%.

To check the stationarity of the variables, we used four unit root tests, namely: ADF (Augmented Dickey-Fuller), ERS, by Elliot et al. (1996), and MZ$_a^{GLS}$ and MZ$_t^{GLS}$, suggested by Perron and Ng (1996) and Ng and Perron (2001). As recommended by Ng and Perron, Hall et al.’s (2012) procedure consists in running tests for changes in the parameters of the structural form for all subsamples defined by the estimated break dates in reduced forms. IPCA is calculated by the Brazilian Institute of Geography and Statistics (IBGE) and this is the price index used by the Brazilian National Monetary Council as benchmark for the inflation-targeting regime. The null hypothesis of the tests is that the series is nonstationary, i.e., unit root.
(2001), the choice of the number of lags \((k)\) was based on the modified Akaike information criterion (MAIC), considering a maximum number of lags of \(k_{\text{max}} = \text{int}(12(T/100)^{1/4}) = 13\). As deterministic components, we included constant \((c)\) and a linear trend \((t)\) for the case in which these components were statistically significant. The results in Table 1 show that, in general, we can reject the unit root hypothesis in the Selic rate, inflation, inflation target, and output gap series.

### 4.2 The CBB’s reaction function with constant parameters

Initially, we estimated the CBB’s reaction function by assuming the parameters of this equation are constant. Since inflation and output gap are potentially endogenous variables, interest rate rule (16) is estimated using the IV method, with robustness of the covariance matrix to heteroskedasticity and serial autocorrelation in the residuals. Specifically, we employed the method proposed by Newey and West (1987) with Bartlett kernel and fixed bandwidth to estimate the variance and covariance matrix. The instruments used are a constant term, lags 1-2 of the Selic rate and of the deviation of inflation from the target, lags 2-3 of output gap, nominal exchange rate movement at \(t-1\) \((\Delta E_{t-1})\) and interaction of the squared inflationary gap with output gap at \(t-2\). These instruments imply two overidentification constraints. The validity of these constraints is tested by the J test of Hansen (1982). Additionally, another two tests are utilized, namely: i) Durbin-Wu-Hausmann’s test to test the null of exogeneity of regressors \((\pi_t - \pi^*_t), y_t\), and \((\pi_t - \pi^*_t)^2y_t\); and ii) the Cragg-Donald \(F\) statistic, proposed by Stock and Yogo (2005), to test the null hypothesis that the instruments are weak.\(^9\)

In Table 2, the test results for the different output gap measures indicate we can reject the hypotheses that regressors \((\pi_t - \pi^*_t), y_t\), and \((\pi_t - \pi^*_t)^2y_t\) are exogenous, and that the instruments used in the regressions are weak. Moreover, the J test shows we cannot reject the null hypothesis that the overidentification constraints are satisfied.

The estimates of reaction function (16) parameters are reported in Table 2. Except for \(d_0\), the monetary rule coefficient values were significant at 5% and quite similar, given the different output gap measures. The estimates for the short-term response of the Selic rate to the inflationary gap, \(d_1\), were equal to 0.075 for the HP specification, and 0.106 for the LT specification. For the output gap, the values of coefficient \(d_2\) indicate that the CBB has also reacted to this variable, but less strongly compared to the inflationary gap.

As to the nonlinearity in the CBB’s reaction function, note that coefficient \(d_3\) was positive and statistically different from zero. This suggests the uncertainty over the effects of output gap on inflation has been important in explaining the adjustment of the Selic rate. In addition, the significance of \(d_3\) indicates that, when output gap is positive, the reaction of the Selic rate to the inflationary gap is increasing in relation to current inflation, and is larger for positive inflationary gaps than for negative ones with the same magnitude (in absolute terms).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>HP</th>
<th>LT</th>
<th>QT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_0)</td>
<td>0.034</td>
<td>0.084</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td>(0.081)</td>
<td>(0.079)</td>
</tr>
<tr>
<td>(d_1)</td>
<td>0.075***</td>
<td>0.106***</td>
<td>0.105***</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.033)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>(d_2)</td>
<td>0.022**</td>
<td>0.017*</td>
<td>0.019*</td>
</tr>
</tbody>
</table>

\(^9\) As stressed by Stock and Yogo (2005), the presence of weak instruments may yield biased IV estimators. Thus, by following these authors, the instruments are considered to be weak when the IV estimator bias relative to the OLS estimator bias is larger than some value \(b\) (for instance, \(b = 5\%\)).
Given the nonlinear structure of the reaction function, the long-term response of the Selic rate to the inflationary gap is given by $\frac{\partial i}{\partial (\pi - \pi^*)} = [d_1 + 2d_3 E((\pi - \pi^*)y)]/(1 - \rho_1 - \rho_2)$, where $E(\cdot)$ denotes the sample mean. The results shown in Table 2 indicate that the response of the Selic rate to a one-percentage-point inflationary gap was equal to 6.52 for the HP specification, and 5.27 for the LT specification. This shows that the monetary policy rule has fulfilled the Taylor (1993) principle, i.e., the CBB has risen the Selic rate by enough to augment the real interest rate in response to an increase in the current inflation gap. Note also that the estimates of $\frac{\partial i}{\partial (\pi - \pi^*)}$ are higher than those obtained by Moura and Carvalho (2010), Aragón and Portugal (2010) and Aragón and Medeiros (2013). These works do not take into account the nonlinearity that results from the uncertainty over the slope in the Phillips curve, and they assess the policy conduct for a shorter period of the inflation-targeting regime.

The long-term response of the Selic rate to the output gap, given by $\frac{\partial i}{\partial y} = [d_2 + d_3 E((\pi - \pi^*)^2)]/(1 - \rho_1 - \rho_2)$, shows that the CBB has reacted to the output gap. When we compare the results of the three specifications, we perceive the reaction of the Selic rate was stronger to the output gap calculated with the HP filter and weaker for the output gap obtained from the linear trend model.

Finally, the Selic rate smoothing for different output gap measures was approximately equal to 0.990, 0.988 and 0.984, respectively. This finding is in line with the literature on short-term interest rate smoothing and reflects the adjustment of this policy instrument at discrete intervals and amounts.

### 4.3 The CBB’s reaction function with structural break

An important assumption about the monetary rule described in the previous section is that its parameters have been stable throughout the analyzed period. Here, we relaxed this assumption and investigated the presence of structural breaks in the CBB’s reaction function coefficients. To do that, we used the OLS-based and IV-based methods described in Section 3.2.

Initially, we tested the stability of the parameters of the reduced forms. This procedure is necessary for the application of the structural break tests in the IV method.\(^{10}\) Table 3 shows the structural break test results and the break date estimates for the reduced form of each endogenous variable of reaction function (16) using different output gap measures.\(^{11}\)

\(^{10}\) As suggested by Perron and Yamamoto (2013, 2014), Hall et al. (2012) and Boldea et al. (2012).

\(^{11}\) For all structural break tests, we considered Bai and Perron’s (1998) sequential procedure, we set the maximum number of breaks at 3 and we used a 15% trimming (which implies that each regime has at least 23 observations).
Following Perron and Yamamoto (2013), we used the fixed regressor bootstrapping method of Hansen (2000) to assess the significance of the tests. The test results show there is at least one break in each equation. Except for the variable \((\pi_t - \pi_t^*)^2y_t\), which has only one break, the FT\((2|1)\) test indicates the existence of multiple breaks in reduced forms for the current inflation gap and output gap in their different measures. With respect to the break dates, we verified that the dates are similar across the different output gap measures. Furthermore, we found that 45% of the breaks took place in 2003 and 2008, which were periods of instability and of economic crisis in Brazil.

| Reaction function | Dependent variable | SupF(1)  | FT(2|1)  | FT(3|2)  | Break dates   |
|-------------------|--------------------|---------|---------|---------|---------------|
| (HP)              | \(\pi_t - \pi_t\) | 96.62***| 63.96***| 27.62   | 03:01:05:04   |
|                   | \(y_{HP,t}\)      | 66.07***| 78.83***| 41.78***| 05:03:08:11:10:11 |
|                   | \((\pi_t - \pi_t^*)^2y_{HP,t}\) | 468.1***| 42.44  |         | 03:08         |
| (LT)              | \(\pi_t - \pi_t\) | 98.25***| 52.59***| 35.95***| 03:01:05:01:10:05 |
|                   | \(y_{TL,t}\)      | 66.01***| 93.36***| 64.83***| 04:07:08:11:10:11 |
|                   | \((\pi_t - \pi_t^*)^2y_{TL,t}\) | 470.9***| 44.80  |         | 03:08         |
| (QT)              | \(\pi_t - \pi_t\) | 99.28***| 54.77***| 33.49***| 03:01:05:01:10:05 |
|                   | \(y_{TQ,t}\)      | 57.74***| 91.46***| 60.48***| 04:07:08:11:10:11 |
|                   | \((\pi_t - \pi_t^*)^2y_{TQ,t}\) | 444.7***| 45.12  |         | 03:08         |

Note: *** denotes significance at 1%, ** at 5%, * at 10%. The test was based on the fixed regressor bootstrapping method of Hansen (2000).

Now, we will test the stability of the CBB’s reaction function parameters through the OLS- and IV-based methods. Table 4 displays the test results and break date estimates. Note that in both the OLS- and IV-based methods, the SupF test allows rejecting the null hypothesis of stability of the CBB’s reaction function parameters at a 5% significance level. In turn, the FT(2|1) test indicates the existence of only one structural break. The estimated break date (July 2003) suggests the conduct of the Brazilian monetary policy was different between Arminio Fraga’s and Henrique Meirelles’s and Alexandre Tombini’s administrations. This finding is consistent with those of Medeiros and Aragón (2013).

| Specif. | SupFT(1) | FT(2|1) | Break date |
|---------|----------|-------|------------|
| (HP)    | 75.01*** | 17.42 | 03:07      |
| (LT)    | 77.18*** | 20.40 | 03:07      |
| (QT)    | 78.88*** | 20.21 | 03:07      |

**OLS-based method**

| Specif. | SupFT(1) | FT(2|1) | Break date |
|---------|----------|-------|------------|
| (HP)    | 57.71*** | 17.04 | 03:07      |
| (LT)    | 60.09*** | 21.24 | 03:07      |
| (QT)    | 61.36*** | 21.12 | 03:07      |

**IV-based method**

Note: *** denotes significance at 1%, ** at 5%, * at 10%. The test was carried out by the fixed regressor bootstrapping method of Hansen (2000).

Given the structural break test results, the next step consists of the estimation of reaction functions for two subperiods determined by the break date, namely: 2000:04-2003:07

---

12 For all tests, the number of bootstrap replications was equal to 1000.
(Fraga’s administration) and 2003:08-2013:12 (Meirelles-Tombini’s administration). The IV estimation method was used, taking into consideration reduced-form structural breaks. Table 5 shows the results for the model. In general, we perceive that the estimated coefficients are similar when analyzed for different output gap measures. Specifically, results reveal a weaker reaction of the short-term Selic rate to inflationary gap ($d_1$). For all specifications that consider different output gap measures, coefficient $d_1$ was significant at 1% in Fraga’s administration. Conversely, in Meirelles-Tombini’s administration, only policy rule with $y_{TQ,i}$ yielded a coefficient $d_1$ statistically different from zero. As to the short-term response of the Selic rate to output gap ($d_2$), we observed it was not significant in Fraga’s administration, but it was positive and significant in Meirelles-Tombini’s administration. Results also demonstrate that coefficient $d_3$, which measures the effect of the uncertainty over the tradeoff in the Phillips curve on the Selic rate, increased slightly after the break date.

Table 5 – CBB’s reaction function estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>HP 00:04-03:08</th>
<th>TL 00:04-03:08</th>
<th>HP 00:04-03:07</th>
<th>TL 00:04-03:12</th>
<th>HP 00:04-03:07</th>
<th>TL 00:04-03:12</th>
<th>HP 03:08-03:07</th>
<th>TL 03:08-13:12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
<td>1.265</td>
<td>1.350</td>
<td>1.199</td>
<td>1.363</td>
<td>0.284</td>
<td>0.294</td>
<td>0.292</td>
<td>0.292</td>
</tr>
<tr>
<td></td>
<td>(1.232)</td>
<td>(1.088)</td>
<td>(1.081)</td>
<td>(1.064)</td>
<td>(0.075)</td>
<td>(0.080)</td>
<td>(0.078)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>$d_1$</td>
<td>0.010</td>
<td>0.013</td>
<td>0.026</td>
<td>0.012</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.055)</td>
<td>(0.052)</td>
<td>(0.050)</td>
<td>(0.054)</td>
<td>(0.055)</td>
<td>(0.052)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.0003)</td>
<td>(0.001)</td>
<td>(0.0003)</td>
<td>(0.001)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>1.410</td>
<td>1.723</td>
<td>1.444</td>
<td>1.712</td>
<td>1.410</td>
<td>1.723</td>
<td>1.444</td>
<td>1.712</td>
</tr>
<tr>
<td></td>
<td>(0.129)</td>
<td>(0.125)</td>
<td>(0.125)</td>
<td>(0.121)</td>
<td>(0.129)</td>
<td>(0.125)</td>
<td>(0.125)</td>
<td>(0.121)</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-0.530</td>
<td>-0.735</td>
<td>-0.567</td>
<td>-0.730</td>
<td>-0.530</td>
<td>-0.735</td>
<td>-0.567</td>
<td>-0.730</td>
</tr>
<tr>
<td></td>
<td>(0.104)</td>
<td>(0.095)</td>
<td>(0.095)</td>
<td>(0.089)</td>
<td>(0.104)</td>
<td>(0.095)</td>
<td>(0.095)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>$\partial_i/\partial(\pi - \pi^*)$</td>
<td>2.144</td>
<td>1.977</td>
<td>2.110</td>
<td>1.420</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\partial_i/\partial(y)$</td>
<td>0.558</td>
<td>0.521</td>
<td>0.636</td>
<td>1.520</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adjusted $R^2$: 0.997 0.997 0.997

Note: ***denotes significance at 1%, ** at 5%, * at 10%. Standard deviation (in brackets). † Indicates the relative bias of the IV estimator in relation to the OLS estimator does not exceed 5%.

The importance in taking into account the uncertainty in the Phillips curve is observed in the long-term reactions of the CBB to the inflationary and output gaps. As outlined in Table 5, the CBB’s long-term reaction to inflationary gap ($\partial_i/\partial(\pi - \pi^*)$) decreased after the break date. This indicates that, despite the increase in $d_3$, the weaker response to the inflationary gap forced down the long-term reaction of the Selic rate to inflation. On the other hand, the long-term response to output gap, given by $\partial_i/\partial y$, increased after Meirelles-Tombini’s administration owing to the increase in both coefficients $d_2$ and $d_3$. The Brazilian monetary authority’s closer attention to the excess demand after 2003 was also verified by Aragón and Medeiros (2014).

4.4 Robustness analysis

In this subsection, we deal with the robustness of the results described in the previous section. The objectives of this exercise are: i) to extend Tillmann’s (2011) model to check if those results are maintained when the CBB’s optimization problem and the structural macroeconomic model take the exchange rate into consideration; and ii) to investigate how the CBB has reacted to exchange rate movements. Besides, we seek to estimate a reaction
function in which the monetary authority acts in a forward-looking manner by responding to the deviations of expected inflation from the inflation target.

Regarding the importance of the exchange rate to the conduct of monetary policy, several studies have investigated whether central banks have reacted directly to exchange rate movements. Clarida et al. (1998) reveal that the central banks of Germany and of Japan include the real exchange rate in their reaction functions, although the magnitude of these reactions is negligible. Mohanty and Klau (2004) estimate modified Taylor rules and observe that several central banks of emerging countries (such as Brazil and Chile) respond to exchange rate movements. Lubik and Schorfheide (2007) estimate a DSGE model for Australia, New Zealand, Canada, and the United Kingdom and note that only the central banks of the first two countries respond to the exchange rate. Consonant with Lubik and Schorfheide (2007), Furlani et al. (2010) demonstrate that the CBB does not change the Selic rate in response to exchange rate movements. Mello and Moccero (2009) observe that the monetary policy instrument reacts to exchange rate in Mexico, but not in Brazil, Chile, and Colombia. Aizenman et al. (2011) and Ostry et al. (2012) point out that the central banks of several emerging markets which adopted the inflation-targeting regime respond to exchange rate movements.

Many are the reasons for the monetary authority to show some direct concern for the exchange rate. First, in an economy with some of its debt denominated in foreign currency, exchange rate depreciations may increase debt service, interfere with firms’ and banks’ balance sheets, restrict credit, cause a surge in filings for bankruptcy, and reduce unemployment and aggregate output. Haussmann et al. (2001) and Calvo and Reinhart (2002) underscore that the effects on economic agents’ balance sheets have been the main reason why central banks try to avoid devaluating their currencies in the presence of external shocks. On the other hand, Aghion et al. (2009) develop a theoretical model to demonstrate exchange rate appreciations can reduce firms’ gains and, consequently, their capacity to take loans and make innovations. This would negatively affect long-term output growth, with a greater impact on economies with a less developed financial system. Aizenman et al. (2011) introduce a simple macroeconomic model for assessment of monetary policy in a small open economy. They noticed that a heavy weight on exchange rate volatility in the central bank’s loss function increases the reaction of the policy instrument to exchange rate and may bring welfare gains. These authors also argue that these gains can be higher in emerging economies, in those which export commodities, in those vulnerable to shocks on terms of trade, and in those with a less developed financial system.

To check whether the CBB has reacted to the exchange rate in an environment characterized by uncertainty about the slope in the Phillips curve, we followed the model of Gali and Monacelli (2005) and Gali (2008) for a small open economy and we assumed that the discretionary policy of this policymaker consists in choosing the Selic rate in period $t$ so as to optimize the following loss function:

$$
\min_{\{\pi_t, \pi^*_t\}} \sum_{t=0}^{\infty} \beta^t \left[ \left( \pi_t - \pi^*_t \right)^2 + \lambda_1 y_t^2 + \lambda_2 q_t^2 + \lambda_3 \left( i_t - \bar{i} \right)^2 + \lambda_4 \left( i_t - i_{t-1} \right)^2 \right]
$$

subject to

$$
y_t = E_t y_{t+1} - \sigma_t^{-1} (i_t - E_t \pi_{t+1} - \bar{r}_t) - \sigma_t^{-1} \alpha (1 - \alpha)^{-1} E_t (q_{t+1} - q_t) \quad (25)
$$

$$\pi_t = \beta E_t \pi_{t+1} + (\kappa_t + z_t^j) y_t - \beta \alpha (1 - \alpha)^{-1} E_t (q_{t+1} - q_t) + \alpha (1 - \alpha)^{-1} E_t (q_t - q_{t-1}) + \bar{u}_t \quad (26)
$$

$$q_t = E_t q_{t+1} - (i_t - E_t \pi_{t+1}) + \epsilon_t \quad (27)
$$
where \( \pi_t \) is the inflation measured by the IPCA, \( q_t \) is the real effective exchange rate gap (i.e., the deviation of the natural logarithm of the real effective exchange rate from the HP-estimated trend), \( \lambda_q \) is the relative weight of the real exchange rate gap in the CBB’s loss function, \( E_t(q_{t+1}) \) is the expected real exchange rate for period \( t+1 \), and \( \varepsilon_t \) is a white-noise error term that reflects the impact of other exchange rate determinants (such as risk premium movements).\(^{13}\) As shown in Galí (2008), parameters \( \sigma_\alpha, \kappa_\alpha > 0 \) are convolutions of the structural macroeconomic model parameters and \( \alpha \in [0,1] \) is regarded as a measure of economic openness. From Eqs. (26) and (27), we may note that, given \( E_tq_{t+1} \), an exchange rate depreciation (increase of \( q_t \)) has a direct positive effect on the output gap and inflation. Eq. (28) shows the exchange rate is determined by the uncovered interest rate parity (UIP).\(^{14}\)

The first-order conditions resulting from the optimization of loss function (25) subject to constraints (26)-(28) can be combined to yield the following interest rate rule:

\[
i_t = (1 - \rho_1) \left[ c_0 + c_1 \left( \pi_t - \pi^*_t \right) + c_2 y_t + c_3 \left( \pi_t - \pi^*_t \right)^2 y_t + c_4 q_t \right] + \rho_2 i_{t-1}
\]

(30)

where

\[
c_0 = \dddot{i}; \quad c_1 = \left[ \kappa_\alpha + \alpha(1 + \beta)\sigma_\alpha \right] / \left[ \lambda_q \sigma_\alpha (1 - \alpha) \right]; \quad c_2 = \lambda_y / \left[ \lambda_q \sigma_\alpha (1 - \alpha) \right]; \quad c_3 = 1 / \left[ \lambda_q \sigma_\alpha (1 - \alpha) \right] \theta_i; \quad c_4 = \lambda_q / \lambda_i ; \quad \rho_1 = \lambda_\omega / \lambda_i + \lambda_\omega
\]

Considering a variable inflation and inserting the nominal interest rate into \( t-2 \) and a random shock, \( u_t \), into (30), we obtained the following specification for the reaction function:

\[
i_t = d_0 + d_1 \left( \pi_t - \pi^*_t \right) + d_2 y_t + d_3 \left( \pi_t - \pi^*_t \right)^2 y_t + d_4 q_t + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \nu_t
\]

(31)

where \( d_\tau = (1 - \rho_1 - \rho_2)c_\tau, \tau = 0, 1, 2, 3,4.\)

In addition, we followed Minella et al. (2003), de Mello and Moccero (2009), Aragón and Portugal (2010) and Minella and Souza-Sobrinho (2013) and also estimated a reaction function specification that includes the deviation of inflation expectations from the inflation target. In this case, the monetary policy rule is expressed by:

\[
i_t = d_0 + d_1 \left( \pi^*_{t,t+1} - \pi_t^* \right) + d_2 y_t + d_3 \left( \pi^*_{t,t+1} - \pi_t^* \right)^2 y_t + d_4 q_t + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \nu_t
\]

(32)

where \( \pi^*_{t,t+1} \) is the expected inflation 12 months ahead conditional on the information available at \( t \).\(^{15,16}\)

---

\(^{13}\) We used the series (no. 11752) of the real exchange rate index - IPCA published by the CBB. The exchange rate gap was obtained from the HP filter.

\(^{14}\) We followed some literature references and set the external (exogenous) variables to zero (see, for instance, Bonomo and Brito, 2002; Leitemo and Söderström, 2008).

\(^{15}\) The expected inflation \( \left( \pi^*_{t,t+1} \right) \) concerns the median of inflation forecasts 12 months ahead (inflation accumulated between \( t \) and \( t+11 \)) made by the market and collected by the CBB’s Investor Relations Group (Gerin). For the period between January 2000 and October 2001, the CBB’s survey does not provide direct information about the inflation rate expected for the next 12 months, but it supplies information about inflation expectations for the current and subsequent years. In this case, one follows Carvalho and Minella (2012) and obtains an approximation to \( \pi^*_{t,t+1} \) subtracting the effective inflation value up to the current month from the expectations for the current year and using the expectations for the following year proportionally to the number of remaining months.

\(^{16}\) For the determinants of inflation expectations in Brazil, see Bevilaqua et al. (2008) and Carvalho and Minella (2012).
In the estimation of specifications (31) and (32), we used the output gap calculated with the HP filter and we took into account the endogeneity of inflationary gaps (both current and expected), of output, and of the exchange rate.\footnote{For specification (31), the instruments used were $i_{t-1}, i_{t-2}, \pi_{t-1}, \pi_{t-1}^{*}, \pi_{t-1}^{*}, y_{t}, y_{t-2}, y_{t-3}$, $(\pi_{t-2}^{*})^{2}y_{t-3}$, $y_{t-2}, y_{t-3}$, and a constant. For specification (32), we included instruments $i_{t-1}, i_{t-2}, \pi_{t-1,1+10}, \pi_{t-1,1+9}, \pi_{t-2}, y_{t}, y_{t-2}, y_{t-3}$, $(\pi_{t-2}^{*})^{2}y_{t-3}$, $y_{t-2}, y_{t-3}$, and a constant.} Table 6 shows the structural break tests for the reduced forms of endogenous regressors. In general, the break tests were statistically different from zero, except for the reduced form of the exchange rate in specification (31). In addition, the $F_{T}(1+1)_{T}$ test indicates three structural breaks in the parameters of the equations of $(\pi_{t} - \pi_{t}^{*})$, $(\pi_{t} - \pi_{t}^{*})^{2}y_{HP,t}$, and $y_{HP,t}$ (in specification 32), and two breaks in the equations of $y_{HP,t}$ (in rule 31), $(\pi_{t+1} - \pi_{t}^{*})$ and $(\pi_{t+1} - \pi_{t}^{*})^{2}y_{HP,t}$. Note that these breaks are taken into consideration in the first stage of the IV estimation of reaction functions.

| Reaction function | Dependent variable | $\text{SupF}(1)$ | $\text{SupF}(2|1)$ | $\text{SupF}(3|2)$ | Break dates |
|-------------------|--------------------|-----------------|-----------------|-----------------|-------------|
| (31)              | $\pi_{t} - \pi_{t}^{*}$ | 114.7$^{[***]}$ | 116.5$^{[***]}$ | 35.54$^{[*]}$ | 03:01; 05:02; 08:12 |
|                   | $y_{HP,t}$         | 155.2$^{[***]}$ | 50.64$^{[***]}$ | 20.95 | 06:03; 09:03 |
|                   | $(\pi_{t} - \pi_{t}^{*})^{2}y_{t}$ | 480.8$^{[***]}$ | 175.3$^{[***]}$ | 53.12$^{[***]}$ | 02:08; 04:08; 08:10 |
|                   | $q_{t}$            | 30.62 | - | - | - |
| (32)              | $(\pi_{t+1} - \pi_{t}^{*})$ | 315.7$^{[***]}$ | 36.36$^{[*]}$ | 19.72 | 03:01; 05:02 |
|                   | $y_{HP,t}$         | 143.3$^{[***]}$ | 35.59$^{[***]}$ | 33.35$^{[***]}$ | 03:08; 07:12; 10:01 |
|                   | $(\pi_{t+1} - \pi_{t}^{*})^{2}y_{HP,t}$ | 187.1$^{[***]}$ | 55.69$^{[***]}$ | 31.94 | 02:11; 04:11 |
|                   | $q_{t}$            | 42.85$^{[**]}$ | 24.53 | - | 02:10 |

Note: $^{***}$ denotes significance at 1%, $^{**}$ at 5%, $^{*}$ at 10%. The test was carried out by the fixed regressor bootstrapping method of Hansen (2000).

Table 7 displays the tests for the instability in the parameters of reaction functions (31) and (32). The OLS-based tests indicate it is not possible to reject the hypothesis of one break in both specifications, whereas the IV-based method only identifies one break in reaction function (31). We followed the OLS-based method because it is, in general, more powerful for the identification of the number of breaks than the IV-based method (Perron and Yamamoto, 2013). Bai and Perron’s (1998) sequential method estimated a break in 2003:07 for specification (31), and a break in 2002:10 for specification (32). Once again, these dates allow classifying the conduct of the Brazilian monetary policy into two administrations: Fraga’s and Meirelles-Tombini’s.

| Specif. | $\text{SupF}(1)$ | $\text{SupF}(2|1)$ | $\text{SupF}(3|1)$ | $T_{1}$ |
|---------|-----------------|-----------------|-----------------|--------|
| (31)    | 85.69$^{[***]}$ | -               | -               | 03:07  |
| (32)    | 50.65$^{[**]}$  | -               | -               | 02:10  |
| (31)    | 79.33$^{[*]}$   | -               | -               | 03:07  |
| (32)    | 30.49           | -               | -               | -      |

Note: $^{***}$ denotes significance at 1%, $^{**}$ at 5%, $^{*}$ at 10%. The test was carried out by the fixed regressor bootstrapping method of Hansen (2000).

Table 8 shows the estimates for reaction functions (31) and (32) obtained by IV for the subperiods determined by the OLS-based method. Initially, the coefficient that implies
nonlinearity for the CBB’s monetary rule, $d_3$, is positive in both specifications and statistically significant at 5% except for the period 2002:11 to 2013:12 in specification (32). In addition, it should be noted that the response of the interest rate to the exchange rate was positive and significant at 1% in specification (31) and significant at 5% in the second subperiod in specification (32). This is consistent with the evidence gathered by Soares and Barbosa (2006), who obtained a positive response of the Selic rate to the real exchange rate, and by Palma and Portugal (2014), who show that the CBB has given a positive weight to the real exchange rate in its loss function.

As to the reactions to inflation and to output, the results indicate that, for specification (31), the long-term response to the inflationary gap decreased, while that of the interest rate to output gap increased. These findings are similar to those described in Section 4.3 for a monetary rule in a closed economy. For specification (32), long-term responses indicate that both the reaction to the expected inflation gap and the reaction to the output gap increased in Meirelles-Tombini’s administration. This suggests that, in this administration, the CBB was more forward-looking, i.e., more concerned about the deviations of expected inflation than about the deviation of current inflation from the target.

5 Conclusion

This paper’s overall aim was to check for the presence of nonlinearities in the CBB’s reaction function as a consequence of this policymaker’s concern about specification errors in the macroeconomic model. The specific aims were: i) to obtain a nonlinear optimal monetary rule that could be robust to the uncertainties over the potential output and the effects of output gap on inflation; ii) to verify the existence of nonlinearities in the CBB’s interest rate rule caused

<table>
<thead>
<tr>
<th>Parameters</th>
<th>00:04-03:07</th>
<th>03:08-13:12</th>
<th>00:04-02:10</th>
<th>02:11-13:12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
<td>1.475</td>
<td>0.110</td>
<td>3.794</td>
<td>0.070</td>
</tr>
<tr>
<td>(1.506)</td>
<td>(0.055)</td>
<td>(2.419)</td>
<td>(0.061)</td>
<td></td>
</tr>
<tr>
<td>$d_1$</td>
<td>0.272</td>
<td>0.002</td>
<td>0.202</td>
<td>0.080</td>
</tr>
<tr>
<td>(0.078)</td>
<td>(0.011)</td>
<td>(0.146)</td>
<td>(0.033)</td>
<td></td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.032</td>
<td>0.026</td>
<td>-0.136</td>
<td>0.026</td>
</tr>
<tr>
<td>(0.080)</td>
<td>(0.006)</td>
<td>(0.111)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>$d_3$</td>
<td>0.002</td>
<td>0.003</td>
<td>0.009</td>
<td>0.001</td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.0003)</td>
<td>(0.002)</td>
<td>(0.0004)</td>
<td></td>
</tr>
<tr>
<td>$d_4$</td>
<td>0.015</td>
<td>0.010</td>
<td>0.005</td>
<td>0.007</td>
</tr>
<tr>
<td>(0.006)</td>
<td>(0.003)</td>
<td>(0.012)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>1.283</td>
<td>1.687</td>
<td>1.242</td>
<td>1.691</td>
</tr>
<tr>
<td>(0.133)</td>
<td>(0.045)</td>
<td>(0.184)</td>
<td>(0.081)</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-0.414</td>
<td>-0.698</td>
<td>-0.462</td>
<td>-0.701</td>
</tr>
<tr>
<td>(0.074)</td>
<td>(0.043)</td>
<td>(0.096)</td>
<td>(0.080)</td>
<td></td>
</tr>
<tr>
<td>$\partial i / \partial (\pi - \pi^*)$</td>
<td>1.918</td>
<td>-0.293</td>
<td>0.913</td>
<td>8.245</td>
</tr>
<tr>
<td>$\partial i / \partial (\gamma)$</td>
<td>0.612</td>
<td>3.716</td>
<td>-0.543</td>
<td>2.864</td>
</tr>
</tbody>
</table>

Adjusted $R^2$ | 0.998 | 0.997

Note: *** denotes significance at 1%, ** at 5%, * at 10%. Standard deviation (in brackets). † Indicates that the relative bias of the IV estimator compared to the OLS estimator does not exceed 5%.
by specification errors; and iii) to run structural break tests to assess possible changes in the CBB’s reaction function coefficients during the inflation-targeting regime.

Due to the presence of potentially endogenous regressors in the reaction function, the method used to test structural breaks in the CBB’s reaction function parameters was developed by Perron and Yamamoto (2013). The procedure proposed by these authors is based on the estimation of the model by OLS and IV, which allows estimating the structural break dates and running tests to verify whether these breaks are statistically significant.

The structural break test results strongly rejected the null hypothesis of stability in the CBB’s reaction function parameters. In general, we observed the presence of a structural break in the third quarter of 2003. This indicates the conduct of the Brazilian monetary policy in Arminio Fraga’s administration was different from that in Henrique Meirelles’s, and also different from Alexandre Tombini’s.

Moreover, empirical evidence points out that the uncertainties over the slope in the Phillips curve implied nonlinearities in the CBB’s reaction function. Specifically, results suggest that when the output gap is positive, the response of the Selic rate to inflationary gap is increasing compared to the current inflation, and is larger for positive inflationary gaps than for negative ones. We also observed an increase in the response of the Selic rate to output gap and a reduction in the reaction to the current inflation gap in Meirelles-Tombini’s administration. Finally, empirical evidence also suggests the CBB has reacted to the exchange rate during Meirelles-Tombini’s administration.

References