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OPTIMAL ECONOMIC GROWTH AND ENVIRONMENTAL ECONOMICS: A BRIEF SUMMARY OF THE LITERATURE

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RESUMO:

Examina em nível teórico, os modelos tradicionais de crescimento econômico, tomando como paradigma o modelo neoclassico de Robert Solow. A finalidade é mostrar que, dados os problemas ecológicos gerados por uma sociedade capitalista moderna, este tipo de modelo passa ao largo de problemas essenciais da realidade atual. Dois modelos que incorporam problemas de poluição e uso de recursos naturais são então propostos, com o alerta de que a natureza mecanicista das técnicas de controle ótimo, mesmo incorporando estas novas características, pode ainda desconsiderar problemas institucionais e políticos.

PALAVRAS-CHAVE:

Crescimento Econômico; Modelo Rober Salow; Economia ambiental; Recursos Naturais; Brasil.

1 INTRODUCTION

This paper examines the contributions of an important theoretical vein of the natural resources and environmental economics literature on optimal economic growth. It is well known that standard neoclassical growth models do not take into account two important characteristics of modern capitalist economies: i) the limited existence and the depletable character of nonrenewable resources used in the economic activities, and ii) the by-product waste generation (e.g., pollution) caused by those activities. Regarding the former, Anderson (1972, p. 256) asserts that " Past analyses of optimal growth behavior have generally ignored the effects of depletion of nonrenewable stocks of productive inputs......The factors of production have been assured to be either self-regenerating (labor) or augmentable via production (capital)". With respect to the latter Foster (1973, p. 554) says that " These (modern growth) theories implicitly assumes no wastes are produced by the economic process, or, alternatively (and more likely), that if any wastes are generated they can be disposed of at no cost to the community......any theory of optimal economic growth that does not take into account these spillovers effects cannot claim to be complete. "

Believing that those concerns are of crucial relevance to address economic growth, this paper takes such a direction and surveys the literature on optimal growth models in order to visualize alternative ways to deal with economic growth in a more complete fashion. To seek that goal, the analysis will consider natural resources and environmental issues as important missing pieces of the traditional approach. To start with, section 2 presents the neoclassical growth model both in its original formulation as in Solow (1956) and optimal control representation as in Rebelo (1991). Also, some critical comments are given regarding the incomplete treatment of the standard approach on the alluded to above missing pieces. Section 3 presents two classes of optimal economic growth models considering, in one perspective, finite and depletable resources and in another, waste generation. In the first class, the optimal growth model of Anderson (1972) will be examined and in the second, the optimal growth model with pollution controls of Forster (1973) will be analyzed. Needless to say, those models make use of a mathematical method called 'optimal control theory' to address the issues on economic growth. Finally, conclusions are presented in the last section.

2 THE NEOCLASSICAL OPTIMAL GROWTH MODELS AND ITS LIMITATIONS

In this section Solow's model will be briefly summarized, and one of its modern representation, using an optimal control method, will be presented (Rebelo, 1991). Close attention will be given to the incomplete character of those models to address economic growth pertinently.

Consider a closed economy that combines capital K_t and labor L_t to produce a homogeneous product. Assume that K_t can be accumulated but L_t cannot and it grows at an exogenous constant rate $n = L_t/L_t$. The production function is the well-known neoclassical Cobb-Douglas with all its standard assumptions:

$$(1)\ Y_t = K_t^{\beta} L_t^{\alpha}, \quad 0 < \alpha < 1 \text{ and } 0 < \beta < 1.$$

Define $\dot{k}_\tau = dK_t/dt$ to be aggregate net investment. Since we assume a closed economy, it must be equal to savings minus depreciation. Assuming a fixed savings rate s, we can write:

$$(2)\ k_\tau = s\, K_t^{\beta}\, L_t^{\alpha} - \delta\, K_t,$$

where δ is the constant depreciation rate applied to capital. Also, define $\kappa_t = K_t/L_t$, i.e., κ_t is the capital/labor ratio. Taking derivatives of κ_t with respect to time we can rewrite (2) in per capita terms as[1]:

$$(2')\ \dot{k}_\tau = s\, \kappa_t^{\beta}\, L_t^{\alpha+\beta-1} - (\delta + n)\kappa_t .$$

We define as steady-state (SS) equilibrium, the situation where all variables grow at a

[1] For details of the mathematical derivation, see Romer (1995).

constant (possibly zero) rate. The growth rate of capital per worker is $\gamma t = \dot{k}\tau/k\tau$, which is constant in steady-state. Dividing both sides of (2') by κt , applying logarithms and then time derivative, we obtain:

(3) $0 = (\beta - 1)\,\gamma t + n\,(\alpha + \beta - 1)$.

Assuming constant returns to scale (CRS), i.e., $\alpha + \beta = 1$, in line with Solow (1956), but decreasing returns to capital ($\beta < 1$), we can rewrite (3) as:

(3') $0 = (\beta - 1)\gamma t$.

But, because $\beta < 1$, the only sustainable SS-growth rate is $\gamma t = 0$. Thus, this model is missing an important piece to explain the well-known long-run growth evidenced by the empirical record. In order to explain long-run growth, neoclassical models of growth reconsider (1) as follows:

(1') $Y_t = A_t\, K_t^{\beta}\, L_t^{\alpha}$,

where $At = Ao.e^{gt}$ is the level of technology that grows at a constant rate g, i.e., g is the exogenous productivity growth rate. This refinement allows the model to tell us that the economy becomes more productive over time as a result of some exogenous technological process. The positive (constant) long-run growth rate of per capita income yt/yt equals g = at/at, and neither a change in household behavior nor government policy can affect it. Therefore, the model explains growth in per capita income along transition paths toward a SS-state, through increases in κt, the capital/labor ratio. High per capita output growth is associate with high investment growth (capital accumulation) relative to labor force growth, which is assumed constant.

Before representing this model as an optimal control problem, we shall ask some questions related to its defects: what about natural resources, which we know, are being used in the economic process? Is this economy an useful one in terms of addressing economic growth by only considering capital and labor as inputs? And, what about the by-product waste generation, consequence of the economic activities in the real world? Clearly, the standard neoclassical model is not able to address those questions. Before presenting the natural resources and environmental economics versions of the growth model, let us first to present its optimal control representation.

Following Rebelo (1991), the optimal control problem is setup via use of a distinct neoclassical Cobb-Douglas production function:

(i) $Y_t = A.K_t$,

where A is an exogenous constant representing technological progress and Kt is aggregate capital as before. Note that this production function is still a Cobb-Douglas with constant returns to scale and constant return to capital. Capital in this case is broadly defined, including not only physical, but also human capital and stock of knowledge. It can be said that there is no role for labor, a nonreproducible resource. The argument is that what is relevant is quality adjusted labor, i.e., human capital is accumulated as each generation is more knowledgeable then the one before.

Using the intensive-form of the above production function to represent the production side of the economy ($yt = A.\kappa t$) and a constant elasticity of substitution (CES) utility function to represent the consumption side, we can formulate the neoclassical growth model as one of optimal control:

(ii) Max $U(ct) = \int_0^{\infty} 0\,(\,_{ct}1 - \alpha - 1/1 - \alpha)\, e^{-rt} dt$

subject to

$\dot{k}\tau = A\,k\tau - \chi\tau$,

where the assumption of St = It is being used, depreciation $\delta = 0$ and $0 < \alpha < 1$ represents the parameter of intertemporal substitution in consumption, r is the discount rate and ct per capita consumption. It should be said that the objective is to maximize the present value of the CES utility function, which has as argument the control (policy) variable ct, subject to the capital accumulation equation, which in turn, explicitly connects the state variable κt to the control variable ct. Obviously, optimal paths of ct will drive the state variable κt in the right optimal path to maximize the present value of the objective functional utility. Note that because this problem is a dynamic optimization involving infinite horizon time framework, transversality conditions, i.e., terminal point constraints do not have a role to play[2].

To setup the maximum principle conditions, we need first to state the current Hamiltonian:

$$\text{(iii)}\ H^{C} = ct^{1-\alpha} - 1 / 1 - \alpha + \mu t\,(A\,\kappa t - ct).$$

The conditions are the following:

$$\partial H^{C}/\partial ct = 0$$

$$\text{(iv)}\quad \dot{\mu} t = \mu t\, r - \partial H^{C}/\partial \kappa t$$

$$\dot{k}t = Ak\tau - \chi\tau.$$

To determine the optimal per capita consumption growth rate is straightforward. Taking logarithms and time derivative of the result in the first condition and using the result of the second in (iv) yields:

$$\text{(v)}\ \dot{c}t/ct = A - r/\alpha$$

This optimal per capita consumption growth is positive as long as $A > r$ (because $0 < \alpha < 1$), i.e., as long as the discount rate is less than the exogenous technological parameter.

Per capita capital growth rate is also easily derived using the third condition in (iv). To see, just apply logarithms and time derivative to both sides to yield:

$$\text{(vi)}\ \dot{k}t/kt = \dot{c}t/ct.$$

Therefore, the long-run per capita capital growth rate is the same as that of optimal per capita consumption, and it is a positive constant as long as the discount rate r is less than the exogenous technological parameter A. Thus, this is what is required by the specification of the optimal control problem: the optimal paths of consumption (the control variable) drives that of capital, which in turn maximizes the present value of utility, given the relevant parameters of the model. It is clear that in the long-run all positive rates of growth must be equal and constant. It should be emphasized that the optimal path for the control variable ct (and thus its growth rate) drives the state variable κt over its optimal path, resulting in the maximum of the present value of the aggregate social utility U(ct).

The derivation of per capita output growth rate can be done similarly. Using (i) in its intensive-form and taking logarithms and time derivative from both sides gives:

$$\text{(vii)}\quad \dot{y}t/yt = \dot{k}t/kt.$$

Therefore, since $\dot{k}t/kt = \dot{c}t/ct$ we have yt/yt = $\dot{y}t/yt = \dot{k}t/kt = \dot{c}t/ct = A - r/\alpha = \lambda_0$. Thus, by representing the neoclassical growth model as an optimal control problem, we have reached the same results as before with the advantage of allowing for the representative agent's optimizing behavior being considered. Interesting results come from this optimal growth model. Firstly, it is possible to determine the optimal savings rate as a fraction of aggregate per capita output in the following way:

$$\text{(viii)}\quad st/yt = \dot{k}t/kt.kt/kt = \lambda_0.(1/A),$$ since $yt = A.\kappa t$ and $\kappa t = It = St$.

Thus, λo (which is the growth rate of per capita output, consumption and capital) is equal to:

$$\text{(ix)}\ \lambda o = A.(st/yt),$$

which can be considered as the growth rate of the economy investigated.

It is worth to note that from (viii) we can rewrite the saving rate as depending on r (discount rate), A (technological parameter) and α (intertemporal substitution parameter), because

[2] For a detailed analysis of dynamic optimization using optimal control theory, see Ching (1992), chapters 7, 8.

we showed before that $\lambda o = A - r / \alpha$. Thus, it is straightforward to see that the more patient a country is (low r), the larger the saving rate and thus its growth rates. This is also true, the more willing a country is to substitute intertemporally(low α).

Having presented the neoclassical growth model both in its original formulation and optimal control frame, we can now address the questions asked before. Would it be feasible to find an optimal path for consumption and capital over time to maximize the present value of welfare represented by the objective functional utility of the economy's agents without any consideration of resources uses and waste generation, both being an unquestionable reality of our days? The answer seems to be in the negative, since the standard approach does not take into account neither of those issues. Next section will present optimal growth models incorporating both resources uses and waste generation.

3 OPTIMAL GROWTH MODELS, NATURAL RESOURCES USES AND WASTE GENERATION

Two classes of models will be analyzed in this section: i) optimal growth with finite and depletable resources and ii) optimal growth with pollution as waste generation. The first model explores the implications of accounting explicitly for the depletion of nonreproducible resources, such as mineral deposits and fossil fuel reserves. The analysis of the optimal growth problem is undertaken by following the standard procedure of making use of a neoclassical, one-sector economy, where the main objective is to find an optimal path of capital accumulation which maximizes the present value of per capita consumption over a finite planning horizon, subject to some specific terminal conditions on the stocks of capital and natural resources.

3.1 AN OPTIMAL CONTROL GROWTH MODEL WITH DEPLETABLE RESOURCES USES

It is important to note that the introduction of a depletable resource into the optimal growth problem modifies the nature of the optimal control procedure, for the infinitely time-period horizon used earlier in the standard growth model is no longer applicable. The implications of this modification lead to a great complexity in terms of the mathematical techniques to be used, but the essence of the problem remains the same as before. Formally, the problem of the first model is formulated by assuming a distinct Leontief production function of the following type:

$$\text{(I)}\quad Yt = \text{Min}\left[F(k\ \ t, Lt), \dot{z}t.e^{\alpha t}\right],$$

where F(.) is the standard neoclassical production function, Yt, the rate of output, Kt, the stock of capital, Lt, input labor, and zt is the stock of depletable resources. Needless to say, $\dot{z}t = dzt/dt$. α is the relative rate of technological progress in resource requirements. From equation (I), if $F(.) < zt.e^{\alpha t}$, we will have:

$$\text{(II)}\ Yt = F(.)\quad \text{and}$$

$$\text{(II')}\quad \dot{z}.t = - e^{-\alpha t}.F(.).$$

Equation (II) tells us that the rate of output Yt is a function of capital and labor over time and equation (II') states that the rate of resource depletion is inversely proportional to the rate of output production, but the proportion diminishes as time passes due to exogenous technological advances (increasing α) that permit depletable resources to be used more efficiently.

The saving-investment identity, i.e., our earlier standard equation for capital accumulation, is the following:

$$\text{(III)}\quad \dot{K}t = st.F(.) - \delta Kt,$$

where 0<st<1 is the savings ratio and δ is the rate of capital depreciation. Now, the optimal growth problem is to find the optimal path for st (the control variable) that maximizes the following present value of consumption over the planning horizon [0, T]:

$$\text{(IV)} \quad \int_0^{\tau} [1 - st].F(.)/Pt.e^{-\mu t}.dt,$$

where Pt is the rate of population and μ is the discount rate. We can rewrite (IV) in its intensive form, as we did before under the standard neoclassical growth model. To do that, it is needed just to assume that population and input labor grow according to $Pt = P_O.e^{\pi t}$ and $Lt = L_O.e^{nt}$, respectively. Thus, the optimal growth problem is the following:

$$\text{(V) Max} \int_0^{\tau} [(1 - st)f(\kappa t)].e^{-rt}.dt,$$

subject to

(i) $\dot{k}t = st.f(\kappa t) - \eta\kappa t$

(ii) $\dot{z}t = -f(\kappa t).e^{-\gamma t}$

(iii) $0 < st < 1$, $\kappa t > 0$, $zt > 0$

(iv) Relevant transversality conditions[3],

where $r = \mu + \pi - n$ is the new discount rate, $\eta = \delta + n$ and $\gamma = \alpha - n$, and all are strictly positive. It is also clear that (1 - st) is per capita consumption and $f(\kappa t)$ is the intensive form of the neoclassical production function. Thus (i) is the equation of capital accumulation in its intensive form and (ii) is the new version of (II') above. The next step is to setup the current Hamiltonian. The two relevant constraints are (i) and (ii), which lead to a problem with two costate variables λt and mt. In this context, these two costates are the shadow price of capital stock and depletable resource, respectively, and thus, they are somewhat similar to lagrangian multipliers. The current Hamiltonian is the following:

$$\text{(VI)} H^C = (1 - st)f(\kappa) + \lambda t[st.f(\kappa t) - \eta\kappa\tau] + mt[-f(\kappa t).e^{-\gamma t}].$$

Clearly, this is a current Hamiltonian similar in character to that already derived, the only difference being the additional depletable resource constraint in the very last part of (VI) and the new end-point restrictions. Because of the necessity of considering the transversality conditions, to maximize H^C at each point in time with respect to st, we need the following decision rules:

(VII) If $\lambda t > 1$, set $st = 1$.

If $\lambda t = 1$, set $st \in [0, 1]$.

If $\lambda t < 1$, set $st = 0$.

In addition, we need the standard conditions of the maximum principle. To get them, we need λt and mt equations, which are given by:

$$\text{(VIII)} \quad \dot{\lambda}t = \lambda t.r - \partial H^C/\partial\kappa t,$$

$$\dot{m}t = mt.r - \partial H^C/\partial zt.$$

Taking the partial derivatives of H^C with respect to the relevant variables and plugging them into (VIII) yields:

$$\text{(IX)} \quad \dot{\lambda}t = [(r + \eta) - st.f'(\kappa t)]\lambda t - [(1 - st).f'(\kappa t) - mt.f'(\kappa t)e^{-\gamma t}],$$

$$\dot{m}t \quad mt.r.$$

Now, using the decision rules stated by (VII) above, and taking into account the conditions in (IX), noting that st can be eliminated from the first equation in (IX) and the relevant equation

[3] The set of transversality conditions involves a complex mathematical procedure that it is not feasible to treat here. For a detailed analysis on optimal control problems with several constraints and end-point transversality conditions, see Chiang (1992), chapter 10.

for the motion of capital accumulation, we can derive the two relevant loci $\lambda t = 0$ and $\kappa t = 0$ using the following conditions:

$$\dot{\lambda}_t = m_o . f'(\mathrm{kt}) e^{(r-y)t} + \begin{cases} [r+\eta-f'(k\tau)]\lambda_t, \ for \lambda_t > 1 \text{ and } s_t = 1 \\ [r+\eta-f'(k\tau)], \text{ for } \lambda_t = 1 \text{ and } s_t \varepsilon \ [0,1] \\ [(r+\eta)\lambda\tau - f'(k\tau)], \text{ for } \lambda_t < 1 \text{ and } s_t = 0 \end{cases}$$

(X)

$$\dot{k}_t = \begin{cases} f(k_t) - \eta k_t, \text{ for } \lambda_t > 1 \text{ and } s_t = 1 \\ s_t . f(k_t) - \eta k_t, \text{ for } \lambda_t = 1 \text{ and } s_t \varepsilon [0,1] \\ -\eta k_t, \text{ for } \lambda_t < 1 \text{ and } s_t = 0. \end{cases}$$

In spite of the apparent complexity, those conditions are quite easy to understand in terms of drawing a phase-diagram in the (λt, κt)-space. Together with the end-point transversality conditions, we can visualize the optimal behavior for capital κt and its shadow price λt. The optimal control analysis can be used to show that when consumption of nonreproducible stocks of resources is considered into the problem, the result is a tendency to postpone capital accumulation and to spend time on growth paths where capital is used less intensively than in a model of unconstrained resources uses.[4] Therefore, the basic result coming from this optimal growth model accounting for depletable resources uses, points out to a general slowdown trend of the economy's growth pace. This is so because the resource constraint poses a limiting restriction on the use of the considered resources, which leads to a reduced rate of capital accumulation and increased rate of savings, which, while acting as the control variable, drives per capita consumption downwards. It should be emphasized that this behavior is the optimal one, in terms of maximizing the present value of the consumption stream stated earlier over time. Therefore, it is optimal to slowdown the economy's capital accumulation when depletable natural resources are considered.

3.2 AN OPTIMAL CONTROL GROWTH MODEL WITH WASTE GENERATION

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The second model of optimal control deals with an important feature not considered by the standard neoclassical growth model. Following Forster (1973), we now present an optimal capital accumulation model taking into account the possibility of waste generation. Specifically, the model will examine the effects of explicitly introducing pollution into the neoclassical growth model developed in section 2. According to Forster (1973, p. 544), " It is naive to think that no wastes are produced and fairly obvious that the free disposal assumption of the neoclassical growth model is not satisfied in the real world."

Making use of the usual procedure, we start with assuming a standard neoclassical production function of the following form:

(a) $Yt = F(Kt)$.

Once again, it is assumed that this production function is well-behaved, in the sense that all standard characteristics apply. It is also assumed that the labor force is a constant proportion of a constant population. The produced output can be either consumed (Ct), invested in capital stock (It) or in pollution control (Et). Therefore, an additional restriction must be imposed in the following way:

(b) $Yt = F(Kt) > Ct + It + Et$.

The usual equation for capital accumulation is thus stated, and δ is the rate of capital depreciation as before:

(c) $\dot{K}_t = It - \delta Kt$.

At this stage we have already the major equations to setup the optimal control problem, but it is reasonable to suppose that capital also produces pollution in addition to physical output. It is also worthy noting that by devoting output to pollution control, the community can lower the amount of pollution generated. Therefore, we can formulate an equation for pollution determination in the following manner:

[4] For the complete description of the phase-diagramatical representation, see Anderson (1972, pp. 261-262).

(d) $Pt = P(Kt, Et)$[5],

where $\partial P/\partial Kt > 0$, $\partial^2 P/\partial Kt^2 > 0$, $\partial P/\partial Et < 0$ and $\partial^2 P/\partial Et^2 > 0$. Finally, the last equation to consider in order to setup the optimal control problem is the linearly separable utility function, assumed to be a function of consumption Ct and pollution Pt, in the following way:

(e) $U(Ct, Pt) = U1(Ct) + U2(Pt)$,

where the marginal utility of consumption is positive but diminishing as usual, and the marginal utility of pollution is negative and decreasing. Now, we are ready to state the optimal control problem. The objective is to maximize the discounted flow of utility over an infinite time horizon. Formally, the problem is to find an optimal path for the relevant variables in order to:

(f) $\text{Max} \int_0^\infty U(Ct, Pt).e^{-rt}\,dt$

subject to

(i) $\dot{k\tau} = It - \delta Kt$, Ko given

(ii) $Pt = P(Kt, Et)$, $Pt > 0$

(iii) $F(Kt) > Ct + It + Et$, $Et > 0$.

To analyze the solution for this problem, we need to formulate the current Hamiltonian, which in this case is as follows:

(g) $H^C = U(Ct, Pt) + \lambda t[It - \delta Kt] + mt[F(Kt) - Ct - It - Et] + \varphi tEt + \theta tPt$.

Again, λt is the shadow-price of capital. Once again, we have a similar problem as the one we derived in the last model of optimal capital accumulation in the presence of depletable resources, the only difference being the very last two terms in (g) and the fact that transversality conditions do not have a role to play, given the infinite-horizon feature of this model. The derivation of the optimal conditions for this model leads to the following equations of motion for consumption and capital accumulation:

(h) $\dot{C}_t = U1'/U1''[r + \delta - \partial P/\partial Kt \,/\, \partial Pt/\partial Et - F'(Kt)]$;

$\dot{K}_t = It - \delta Kt$.

Using those equations we can investigate the behavior of the capital stock in the (Kt , Ct)-space, in a somehow mirrored manner we mentioned earlier.[6] The relevant results coming from this optimal control model point out that when pollution is accounted for, the economy tends to a lower capital stock accumulation, than when pollution is not considered, the same qualitative result attained in our earlier analysis of the depletable resource model. It should be said that, despite the simplicity of the present model, the results strongly suggest that the standard neoclassical growth model is biased in its results.

Having presented the two classes of optimal growth models accounting for environmental issues, in one hand, considering depletable resources, and in the other, pollution as waste generation, it is time to evaluate those alleged refinements as improvements upon the standard neoclassical model. Surely, at least in terms of considering the introduction of those environmental issues, the models discussed above seem to have their relevance, as compared with the standard growth model. But, it is true that depletable resources, pollution generation, output production and consumption are all interrelated issues, and thus, to be fully complete such models would have to consider all of them at the same time. Also, a more serious problem is that those models bring about a set of weakness in their formulations. Firstly, there is an important internal difficulty related to the use of a given discount rate, issue which authors rarely discuss. It is very hard to find an appropriate social discount rate to perform the calculations involved in those optimal control problems, and thus, empirical work on this theme poses a lot of challenges and, at the same time, difficulties. Another set of criticisms refers to the formal and mechanistic manner upon which optimal control models are based. To deal with environmental issues in a pertinent way, political and institutional framework must play a very

[5] Note that there is no stock accumulation of pollutant in this model, a recognizable shortcoming. But, it can be easily introduced without substantial changes. See Foster (1993) for this extention.

[6] The detailed phase-diagramatical and mathematical analyses are presented in Forster (1973, pp. 546-547).

important role, a feature that the formal analysis of optimal control theory is far to acquire.

4 CONCLUSION

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This paper's objective was to sinthetize and examine critically the literature on optimal growth models, firstly considering the standard neoclassical growth version and then allowing for the introduction of environmental issues as relevant variables determining the character of optimal economic growth. It was seen that the neoclassical growth model cannot be considered complete, because of its missing assumptions on resources uses and waste generation, two characteristics of modern economic systems. The two chosen models examined above in section 3 took into account those missing pieces and, via optimal control device, provided some insights into the essential nature of economic growth.

It should be emphasized, however, that those efforts must be understood in a restrict sense, since we cannot say they represent unquestionable improvements. It was put that the mechanistic nature of the optimal control theory is not well suited to deal with environmental issues, the reason being that institutional and political action may be much more important to bring into the analysis. Also, there is an additional model's internal difficulty represented by the appropriate social discount rate to be used to calculate present values in those types of models. But, at least as long as we are assured to make a good use of an analytical tool like the optimal control theory, suggestive results may rise. To cite a leading mathematician on this, " After.......so much time and effort to master the various facets of the dynamic-optimization tool (particularly, optimal control theory), we really ought not to end on a negative note. So by all means go ahead and have fun playing with......Hamiltonians, transversality conditions, and phase-diagrams to your heart's content. But do please bear in mind what they can and cannot do for you ". (Chiang, 1992, p. 314).

ABSTRACT:

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This paper examines, at a theorethical level, traditional models of economic growth electing as a standard Robert Solow´s neo-classical model. Its main purpose is to demonstrate that crucial elements such as pollution and waste in the use of natural resources are absent of this paradigm. Two new models are then proposed, incorporating these new traits of modern capitalist societies, with the proviso that, given the mechanistic nature of optimal control theory factors, like institutional background and political interaction, cannot be considered yet in these new models.

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